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# Strategic Financial Management: Exercises

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# About the Author

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With increasing demand for global e-learning, his attention is now focussed on the free provision of a financial textbook series, underpinned by a critique of contemporary capital market theory in volatile markets, published by bookboon.com.

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# Part One
An Introduction

# 1 Finance – An Overview

## Introduction

It is a basic assumption of finance theory, taught as fact in Business Schools and advocated at the highest level by vested interests, world-wide (governments, financial institutions, corporate spin doctors, the press, media and financial web-sites) that stock markets represent a profitable long-term investment. Throughout the twentieth century, historical evidence also reveals that over any five to seven year period security prices invariably rose.

This happy state of affairs was due in no small part (or so the argument goes) to the *efficient* allocation of resources based on an *efficient* interpretation of a *free flow* of information. But nearly a decade into the new millennium, investors in global markets are adapting to a new world order, characterised by economic recession, political and financial instability, based on a *communication breakdown* for which strategic financial managers are held largely responsible.

The root cause has been a breakdown of *agency theory* and the role of *corporate governance* across global capital markets. Executive managers motivated by their own greed (short-term bonus, pension and share options linked to short-term, high-risk profitability) have abused the complexities of the financial system to drive up value. To make matters worse, too many companies have also flattered their reported profits by adopting *creative accounting* techniques to cover their losses and discourage predators, only to be found out.

*We live in strange times*. So let us begin our series of Exercises with a critical review of the traditional *market* assumptions that underpin the Strategic Financial Management function and also validate its decision models. A fundamental re-examination is paramount, if companies are to regain the trust of the investment community which they serve.

## Exercise 1.1: Modern Finance Theory

We began our companion text: *Strategic Financial Management* (*SFM* henceforth) with an idealised picture of shareholders as wealth maximising individuals, to whom management are ultimately responsible. We also noted the theoretical assumption that shareholders *should be* rational, risk-averse individuals who demand higher returns to compensate for the higher risk strategies of management.

What *should be* (rather than *what is*) is termed *normative theory*. It represents the bedrock of modern finance. Thus, in a sophisticated mixed market economy where the ownership of a company's investment portfolio is divorced from its control, it follows that:

> The over-arching, normative objective of strategic financial management should be an optimum combination of investment and financing policies which maximise shareholders' wealth as measured by the overall return on ordinary shares (dividends plus capital gains).

But what about the "real world" of *what is* rather than *what should be*?

A fundamental managerial problem is how to retain funds for reinvestment without compromising the various income requirements of innumerable shareholders at particular points in time.

*As a benchmark*, you recall from *SFM* how Fisher (1930) neatly resolved this dilemma. In *perfect markets*, where all participants can borrow or lend at the same market rate of interest, management can maximise shareholders' wealth irrespective of their consumption preferences, providing that:

> The return on new corporate investment at least equals the shareholders' cost of borrowing, or their desired return earned elsewhere on comparable investments of equivalent risk.

Yet, eight decades on, we all know that markets are *imperfect*, characterised by *barriers to trade* and populated by *irrational* investors, each of which may invalidate Fisher's *Separation Theorem*.

As a consequence, the questions we need to ask are whether an *imperfect* capital market is still *efficient* and whether its constituents exhibit *rational* behaviour?

- *If so,* shares will be correctly priced according to a firm's investment and financial decisions.
- *If not*, the global capital market may be a "castle built on sand".

So, before we review the role of Strategic Financial Management, outlined in Chapter One of our companion text, let us evaluate the case *for and against* stock market *efficiency*, investor *rationality* and summarise its future implications for the investment community, including management.

As a springboard, I suggest reference to Fisher's Separation Theorem (*SFM*: Chapter One). Next, you should key in the following terms on the internet and itemise a *brief* definition of each that you feel comfortable with.

*Perfect Market; Agency Theory; Corporate Governance; Normative Theory; Pragmatism; Empiricism; Rational Investors; Efficient Markets; Random Walk; Normal Distribution; EMH; Weak, Semi-Strong, Strong; Technical, Fundamental (Chartist) and Speculative Analyses.*

Armed with this information, answer the questions below. But keep them brief by using the previous terms at appropriate points *without* their definitions. Assume the reader is familiar with the subject.

Finally, compare your answers with those provided and if there are points that you do not understand, refer back to your internet research and if necessary, download other material.

The Concept of Market Efficiency as "Bad Science"

1. How does Fisher's Separation Theorem underpin modern finance?
2. If capital markets are imperfect does this invalidate Fisher's Theorem?
3. Efficient markets are a necessary but not sufficient condition to ensure that NPV maximisation elicits shareholder wealth maximisation. Thus, modern capital market theory is not premised on efficiency alone. It is based on three pragmatic concepts.

Define these concepts and critique their purpose.

4. Fama (1965) developed the concept of efficient markets in three forms that comprise the Efficient Market Hypothesis (EMH) to justify the use of linear models by corporate management, financial analysts and stock market participants in their pursuit of wealth.

Explain the characteristics of each form and their implications for technical, fundamental and speculative investors.

5. Whilst governments, markets and companies still pursue policies designed to promote stock market efficiency, since the 1987 crash there has been increasing unease within the academic and investment community that the EMH is "bad science".

Why is this?

6. What are your conclusions concerning the Efficient Market Hypothesis?

**An Indicative Outline Solution (Based on Key Term Research)**

1. **Fisher's Separation Theorem**
   In corporate economies where ownership is divorced from control, firms that satisfy consumer demand should generate money profits that create value, increase equity prices and hence shareholder wealth.

   To achieve this position, corporate management must optimise their internal investment function and their external finance function. These are interrelated by the firm's cost of capital compared to the return that investors can earn elsewhere.

To resolve the dilemma, Fisher (1930) states that in *perfect markets* a company's investment decisions can be made independently of its shareholders' financial decisions without compromising their wealth, providing that returns on investment at least equal the shareholders' opportunity cost of capital.

But how perfect is the capital market?

2. **Imperfect Markets and Efficiency**
We know that capital markets are not perfect but are they reasonably efficient? If so, profitable investment undertaken by management on behalf of their shareholders (the *agency principle* supported by *corporate governance*) will be communicated to market participants and the current price of shares in issue should rise. So, conventional theory states that firms should maximise the cash returns from all their projects to maximise the market value of ordinary shares

3. **Capital Market Theory**
Modern capital market theory is based on three *normative* concepts that are also *pragmatic* because they were accepted without any *empirical* foundation.

- *Rational investors*
- *Efficient markets*
- *Random walks*

To prove the point, we can question the first two: investors are "irrational" (think Dot.Com) and markets are "inefficient" (insider dealing, financial meltdown and governmental panic)). So, where does the concept of "random walk" fit in?

If investors react rationally to new information within efficient markets it should be impossible to "beat the market" except by luck, rather than judgement. The first two concepts therefore justify the third, because if "markets have no memory" the past and future are "independent" and security prices and returns exhibit a random *normal distribution*.

So, why do we have a multi-trillion dollar financial services industry that reads the news of every strategic corporate financial decision throughout the world?

4. **The Efficient Market Hypothesis (EMH)**

   Anticipating the need for this development, Eugene Fama (1965 *ff.*) developed the Efficient Market Hypothesis (*EMH*) over forty years ago in three forms (*weak, semi-strong and strong*). Irrespective of the form of market efficiency, he explained how:

   - Current share prices reflect all the information used by the market.
   - Share prices only change when new information becomes available.

   As markets strengthen, or so his argument goes, any investment strategies designed to "beat the market" weaken, whether they are *technical (i.e. chartist), fundamental* or a combination of the two. Like *speculation*, without insider information (illegal) investment is a "fair game for all" unless you can afford access to market information before the competition (i.e. semi-strong efficiency).

5. **The EMH as "Bad Science"**
   Today, despite global recession, governments, markets and companies continue to promote policies premised on semi-strong efficiency. But since the 1987 crash there has been an increasing awareness within the academic community that the EMH in any form is "bad science". It placed the "cart before the horse" by relying on three simplifying assumptions, without any empirical evidence that they are true. Financial models premised on rationality, efficiency and random walks, which are the bedrock of modern finance, therefore attract legitimate criticism concerning their real world applicability.

6. **Conclusion**
   Post-modern behavioural theorists believe that markets have a memory, take a "non-linear" view of society and dispense with the assumption that we can maximise anything with their talk of speculative bubbles, catastrophe theory and market incoherence. Unfortunately, they too, have not yet developed alternative financial models to guide corporate management in their quest for shareholder wealth *via* equity prices.

   So, who knows where the "new" finance will take us?

## Exercise 1.2: The Nature and Scope of Financial Strategy

Although the capital market assumptions that underpin modern finance theory are highly suspect, it is still widely accepted that the normative objective of financial management is the maximisation of shareholder wealth. We observed in Chapter One of our companion text (*SFM*) that to satisfy this objective a company requires a "long-term course of action". And this is where *strategy* fits in.

**Financial Strategy and Corporate Objectives**

Using *SFM* supplemented by any other reading:

1. Define Corporate Strategy
2. Explain the meaning of Financial Strategy?
3. Summarise the functions of Strategic Financial Management.

**An Indicative Outline Solution**

1. **Corporate Strategy**
   *Strategy* is a course of action that specifies the monetary and physical resources required to achieve a predetermined objective, or series of objectives.

   *Corporate Strategy* is an overall, long-term plan of action that comprises a portfolio of functional business strategies (finance, marketing etc.) designed to meet the specified objective(s).

2. **Financial Strategy**

   *Financial Strategy* is the portfolio constituent of the corporate strategic plan that embraces the optimum investment and financing decisions required to attain an overall specified objective(s).

   It is also useful to distinguish between strategic, tactical and operational planning.

   - Strategy is a long-run course of action.
   - Tactics are an intermediate plan designed to satisfy the objectives of the agreed strategy.
     - Operational activities are short-term (even daily) functions (such as inventory control) required to satisfy the specified corporate objective(s) in accordance with tactical and strategic plans.

   Needless to say, senior management decide strategy, middle management decide tactics and line management exercise operational control.

3. **The Functions of Strategic Financial Management**

   We have observed financial strategy as the area of managerial policy that determines the investment and financial decisions, which are preconditions for shareholder wealth maximisation. Each type of decision can also be subdivided into two broad categories; longer term (strategic or tactical) and short-term (operational). The former may be unique, typically involving significant fixed asset expenditure but uncertain future gains. Without sophisticated periodic forecasts of required outlays and associated returns that model the *time value of money* and an allowance for risk, the subsequent penalty for error can be severe, resulting in corporate liquidation.

   Conversely, operational decisions (the domain of working capital management) tend to be repetitious, or infinitely divisible, so much so that funds may be acquired piecemeal. Costs and returns are usually quantifiable from existing data with any weakness in forecasting easily remedied. The decision itself may not be irreversible.

   However, irrespective of the time horizon, the investment and financial decision functions of financial management should always involve:

   - The continual search for investment opportunities.
   - The selection of the most profitable opportunities, in absolute terms.
   - The determination of the optimal mix of internal and external funds required to finance those opportunities.
   - The establishment of a system of financial controls governing the acquisition and disposition of funds.
   - The analysis of financial results as a guide to future decision-making.

None of these functions are independent of the other. All occupy a pivotal position in the decision making process and naturally require co-ordination at the highest level.

## Summary and Conclusions

The implosion of the global free-market banking system (and the domino effect throughout world-wide corporate sectors starved of finance) required consideration of the assumptions that underscore modern financial theory. Only then, can we place the following Exercises that accompany the companion *SFM* text within a topical framework.

However, we shall still adhere to the traditional objective of shareholder wealth maximisation, based on *agency theory* and *corporate governance*, whereby the owners of a company entrust management with their money, who then act on their behalf in their best long-term interests.

But remember, too many financial managers have long abused this trust for personal gain.

So, whilst what follows is a *normative* series of Exercises based on "what should" be rather than "what is", it could be some time before Strategic Financial Management and the models presented in this text receive a good press.

# Part Two
The Investment Decision

# 2 Capital Budgeting Under Conditions of Certainty

## Introduction

If we assume that the strategic objective of corporate financial management is the maximisation of shareholders' wealth, the firm requires a consistent model for analysing the profitability of proposed investments, which should incorporate an appropriate criterion for their acceptance or rejection. In Chapter Two of *SFM* (our companion text) we examined *four common techniques* for selecting capital projects where a choice is made between alternatives.

- *Payback* (PB) is useful for calculating how quickly a project's cash flows recoups its capital cost but says nothing about its overall profitability or how it compares with other projects.
- *Accounting Rate of Return* (ARR) focuses on project profitability but contains serious computational defects, which relate to accounting conventions, ignores the *true* net cash inflow and also the time value of money.

When the time value of money is incorporated into investment decisions using *discounted cash flow* (DCF) techniques based on *Present Value* (PV), the real *economic* return differs from the accounting return (ARR). So, the remainder of our companion chapter explained how DCF is built into investment appraisal using one of two PV models:

- *Internal Rate of Return* (IRR)
- *Net Present Value* (NPV).

In practice, which of these models management choose to maximise project profitability (and hopefully wealth) often depends on how they define "profitability". If management's objective is to maximise *the rate of return in percentage terms* they will use IRR. On the other hand, if management wish to maximise *profit in absolute cash terms* they will use NPV.

But as we shall discover in this chapter and the next, if management's over-arching objective is wealth maximisation then the IRR may be *sub-optimal* relative to NPV. The problem occurs when ranking projects in the presence of *capital rationing*, if projects are *mutually exclusive* and a choice must be made between alternatives.

## Exercise 2.1: Liquidity, Profitability and Project PV

Let us begin our analysis of profitable, wealth maximising strategies by comparing the four methods of investment appraisal outlined above (PB, ARR, IRR and NPV) applied to the same projects.

The Bryan Ferry Company operates regular services to the Isle of Avalon. To satisfy demand, the Executive Board are considering the purchase of an idle ship (the "Roxy") as a temporary strategy before their new super-ferry (the "Music") is delivered in four years time.

Currently, laid up, the Roxy is available for sale at a cost of $2 million. It can be used on one of two routes: either an existing route (Route One) subject to increasing competition, or a new route (Two) which will initially require discounted fares to attract custom.

Based on anticipated demand and pricing structures, Ferry has prepared the following profit forecast ($000) net of straight-line depreciation with residual values and capital costs.

| Route | | One | Two |
|---|---|---|---|
| Pre-Tax Profits | | | |
| Year: | One | 800 | 300 |
| | Two | 800 | 500 |
| | Three | 400 | 900 |
| | Four | 400 | 1,200 |
| Residual Value 400 400 | | | |
| Cost of Capital 16% 16% | | | |

**Required:**

Using this data, information from Chapter Two of the *SFM* text, and any other assumptions:

1. Summarise the results of your calculations for each route using the following criteria.
   Payback (PB); Accounting Rate of Return (ARR);
   Internal Rate of Return (IRR); Net Present Value (NPV)

2. Summarise your acceptance decisions using each model's maximisation criteria.

To answer this question and others throughout the text you need to access Present Value (PV) tables from your recommended readings, or the internet. Compound interest and zstatistic tables should also be accessed for future reference. To get you started, however, here is a highlight from the appropriate PV table for part of your answer (in $).

Present Value Interest Factor ($1 at *r* % for *n* years) = $1/(1+r)^n$

| Factor | 16% |
|---|---|
| Year One | 1.000 |
| Year Two | 0.862 |
| Year Three | 0.743 |
| Year Four | 0.552 |
| Year Five | 0.476 |

**An Indicative Outline Solution**

Your analyses can be based on either four or five years, depending on when the Roxy is sold (realised). Is it at the end of Year 4 or Year 5? These assumptions affect IRR and NPV investment decision criteria but not PB. Even though all three are *cash-based*, remember that PB only relates to *liquidity* and *not profitability*. The ARR will also differ, according to your accounting formula. For consistency, I have used a simple *four-year* formula ($m) throughout. For example, with Route One:

Average Lifetime Profit / Original Cost less Residual Value = 0.6 / 2.0 = 30%

The following results are therefore illustrative but not exhaustive. Your answers may differ in places but this serves to highlight the importance of stating the assumptions that underpin any financial analyses.

**1: Results**

Let us assume the Roxy is sold in *Year Five* (with ARR as a cost-based four-year average).

| Criteria | PB(Yrs) | ARR(%) | IRR(%) | NPV($000) |
|---|---|---|---|---|
| Route 1 | 1.67 | 30.00 | 42.52 | 1,101.55 |
| Route 2 | 2.31 | 36.25 | 38.70 | 1,209.73 |

Now assume the Roxy is realised in Year *Four* (where PB and ARR obviously stay the same).

| Criteria | PB(Yrs) | ARR(%) | IRR(%) | NPV ($000) |
|---|---|---|---|---|
| Route 1 | 1.67 | 30.00 | 41.49 | 1,071.08 |
| Route 2 | 2.31 | 36.25 | 37.88 | 1,179.26 |

**2: Project Acceptance**

According to our four investment models (irrespective of when the Roxy is sold) project selection based upon their respective criteria can be summarised as follows:

| Criteria | PB(Yrs) | ARR(%) | IRR(%) | NPV($000) |
|---|---|---|---|---|
| Objective | (Max. Liq.) | (Max. %) | (Max. %) | (Max. $) |
| Route | 1 | 2 | 1 | 2 |

Unfortunately, if Bryan Ferry's objective is wealth maximisation, we have a dilemma. Which route do we go for?

We can dispense with PB that maximises liquidity but reveals nothing concerning profitability and wealth. The ARR is also dysfunctional because it is an average percentage rate based on accrual accounting that also ignores project size and the time value of money. Unfortunately, this leaves us with the IRR, which favours Route One and the NPV that selects Route 2.

So, give some thought to which route should be accepted before we move on to the next exercise and a formal explanation of our ambiguous conclusion in Chapter Three.

## Exercise 2.2: IRR Inadequacies and the Case for NPV

Profitable investments opportunities are best measured by DCF techniques that incorporate the time value of money. Unfortunately, with more than one DCF model at their disposal, which may also give conflicting results when ranking alternative investments, management need to define their objectives carefully before choosing a model.

You will recall from the *SFM* text that in a free market economy, firms raise funds from various providers of capital who expect an appropriate return from efficient asset investment. Under the assumptions of a perfect capital market, explained in Part One, the firm's investment decision can be separated from the owner's personal preferences without compromising wealth maximisation, providing projects are valued on the basis of their opportunity cost of capital. If the cut-off rate for investment corresponds to the market rate of interest, which shareholders can earn elsewhere on similar investments:

> Projects that produce an IRR greater than their opportunity cost of capital (i.e. positive NPV) should be accepted. Those with an inferior return (negative NPV) should be rejected.

Even in a world of zero inflation, the DCF concept also confirms that in today's terms the PV of future sums of money is worth progressively less, as its receipt becomes more remote and interest rates rise.

This phenomenon is supremely important to management in a situation of *capital rationing*, or if investments are *mutually exclusive* where projects must be ranked in terms of the timing and size of prospective profits which they promise. Their respective PB and ARR computations may be uniform. Their initial investment cost and total net cash inflows over their entire lives may be identical. But if one delivers the bulk of its return earlier than any other, it may exhibit the highest present value (PV). And providing this project's return covers the cost and associated interest payments of the initial investment it should therefore be selected. Unfortunately, this is where modelling optimum strategic investment decisions using the IRR and NPV conflict.

**Required:**

Refer back to Chapter Two of the companion text (and even Chapter Three) and without using any mathematics summarise in your own words:

1. The IRR concept.
2. The IRR accept-reject decision criteria.
3. The computational and conceptual defects of IRR.

**An Indicative Outline Solution**

1. **The IRR Concept**
   The IRR methodology *solves* for an *average* discount rate, which equates future net cash inflows to the present value (PV) of an investment's cost. In other words, the IRR equals the *hypothetica*l rate at which an investment's NPV would equal zero.

2. **IRR Accept-Reject Decision Criteria.**
   The solution for IRR can be interpreted in one of two ways.

   - The time-adjusted rate of return on the funds committed to project investment.
   - The maximum rate of interest required to finance a project if it is not to make a loss.

   The IRR for a given project can be viewed, therefore, as a *financial break-even point* in relation to a cut-off rate for investment predetermined by management. To summarise:

*Individual* projects are acceptable if:

IRR ≥ a target rate of return
IRR > the cost of capital, or a rate of interest.

*Collective* projects can be ranked according to the size of their IRR. So, under conditions of capital *rationing*, or where projects are *mutually exclusive* and management's objective is IRR *maximisation*, it follows that if:

$$IRR_A > IRR_B > \ldots IRR_N$$

Project A would be selected, subject to the proviso that it at least matched the firm's cut-off rate criterion for investment.

3. **The Computational and Conceptual Defects of IRR.**
   Research the empirical evidence and you will find that the IRR (relative to PB, ARR and NPV) often represents the preferred method of strategic investment appraisal throughout the global business community. Arguments in favour of IRR are that

- Profitable investments are assessed using *percentages* which are *universally* understood.
- If the annual net cash inflows from an investment are equal in amount, the IRR can be determined by a simple formula using factors from PV annuity tables.
- Even if annual cash flows are complex and a choice must be made between alternatives, commercial software programs are readily available (often as freeware) that perform the chain calculations to derive each project's IRR

Unfortunately, these practical selling points overstate the case for IRR as a profit maximisation criterion.

You will recall from our discussion of ARR that percentage results fail to discriminate between projects of different *timing and size* and may actually conflict with wealth maximisation. Firms can maximise their rate of return by accepting a "quick" profit on the smallest "richest" project. However, as we shall discover in Chapter Three, high returns on low investments (albeit liquid) do not necessarily maximise *absolute* profits.

When net cash inflows are equal in amount, a factor computation may not correspond exactly to an appropriate figure in a PV annuity table, therefore requiring some method of interpolation. Even with access to computer software, it soon emerges that where cash flows are variable a project's IRR may be *indeterminate*, not a *real number* or with *imaginary roots*.

Computational difficulties apart, conceptually the IRR also assumes that even under conditions of certainty when capital costs, future cash flows and life are known and correctly defined:

- All financing will be undertaken at a cost equal to the project's IRR.
- Intermediate net cash inflows will be reinvested at a rate of return equal to the IRR.

The implication is that inward cash flows can be reinvested at the *hypothetical* interest rate used to finance the project and in the calculation of a zero NPV. Moreover, this borrowing-reinvestment rate is assumed to be constant over a project's life. Unfortunately, relax either assumption and the IRR will change.

## Summary and Conclusions

Because the precise derivation of a project's IRR present a number of computational and conceptual problems, you may have concluded (quite correctly) that a *real* rather than *assumed* cut-off rate for investment should be incorporated directly into present value calculations. Presumably, if a project's NPV based on a real rate is positive, we should accept it. Negativity would signal rejection, unless other considerations (perhaps non-financial) outweigh the emergence of a residual cash deficit.

# 3 Capital Budgeting and the Case for NPV

## Introduction

If management invest resources efficiently, their strategic shareholder wealth maximising objectives should be satisfied. Chapter Two of both the *SFM* and *Example* texts explain the superiority of the NPV decision model over PB, ARR and IRR as a strategic guide to action. Neither PB, nor ARR, maximise wealth. IRR too, may be *sub-optimal* unless we are confronted by a single project with a "normal" series of cash inflows. We concluded, therefore, that under conditions of certainty with known price level movements:

> Managerial criteria for wealth maximisation should conform to an NPV maximisation model that discounts *incremental* money cash flows at their *money* (market) rate of interest.

Chapter Three of *SFM* compares NPV and IRR project decision rules. We observed that differences arise because the NPV is a measure of *absolute money wealth,* whereas IRR is a *relative percentage measure*. NPV is also free from the computational difficulties frequently associated with IRR. The validity of the two models also hinges upon their respective assumptions concerning borrowing and reinvestment rates associated with individual projects.

Unlike NPV, IRR assumes that re-investment and capital cost rates equal a project's IRR without any economic foundation whatsoever and important consequences for project rankings. We shall consider this in our first Exercise.

Of course, NPV is still a financial *model*, which is an *abstraction* of the real world. Select simple data from complex situations and even NPV loses detail. But as we shall observe in our second Exercise, incorporate real-world considerations into NPV analyses (relevant cash flows, taxation, price level changes) and we can prove its strategic wealth maximising utility.

## Exercise 3.1: IRR and NPV Maximisation

The Jovi Group is deciding whether to proceed with one of two projects that have a three-year life. Their respective IRR (highlighted) assuming relevant cash flows are as follows (£000s):

| | Cost | Annual | Net | Inflows | IRR |
|---|---|---|---|---|---|
| Year | 0 | 1 | 2 | 3 | |
| Project 1 | 1,000 | 500 | 700 | 900 | **43 %** |
| Project 2 | 1,000 | 1,000 | 500 | 500 | **54 %** |

**Required:**

Given that Jovi's cost of capital is a uniform 10 percent throughout each project:

1. Calculate the appropriate PV discount factors.
2. Derive each project's NPV compared to IRR and highlight which (if any) maximises corporate wealth according to both investment criteria.
3. Use the NTV concept to prove that NPV maximises wealth in *absolute money terms*.
4. Explain why IRR and NPV rank projects differently using a graphical analysis.

**An Indicative Outline Solution**

Your answer should confirm that individually each project will *increase* wealth because both IRRs exceed the cost of capital (i.e. the discount rate) and both NPVs are positive. But if a choice must be made between the alternatives, only one project *maximises* wealth. And to complicate matters further, NPV maximisation and IRR maximisation criteria rank the projects differently. So, which model should management use?

**1: PV Factor Calculations for $1/(1+r)^t$ (£1 at 10% for t years where t = 0 to 3)**

$1/(1.1)^0 = 1.000$ 1 $1/(1.1)^1 = 0.909$ $1/(1.1)^2 = 0.826$ $1/(1.1)^3 = 0.751$

**2: NPV (£ 000s ) and IRR (%) Highlighted Comparisons**

| | | NPV | IRR |
|---|---|---|---|
| Project 1(10%): (1,000) + 500 × 0.909 + 700 × 0.826 + 900 × 0.751 | = | **709** | 43% |
| Project 2(10%): (1,000) + 1,000 × 0.909 + 500 × 0.826 + 500 × 0.751 | = | 698 | **54%** |

NPV maximisation selects one project but IRR maximisation selects the other; but why?

**3: NTV (£ 000s)**

Assume that Jovi borrows £1 million at an interest rate of 10 per cent to invest in either project but not both. They are *mutually exclusive*. Thereafter, reinvestment opportunities also yield 10 per cent. The *bank overdraft* formulation below reveals that if project funds are reinvested at the market rate of interest, NPV not only favours Project 1 but also *maximises wealth* because it produces a higher *cash surplus* (NTV) at the end of three years.

| **Project ( £000s )** | | **1** | | **2** |
|---|---|---|---|---|
| Cost | - | 1,000 | - | 1,000 |
| Interest year 1 | - | 100 | - | 100 |
| | - | 1,100 | - | 1,100 |
| Receipt year 1 | + | 500 | +- | 1,000 |
| | - | 600 | | 100 |
| Interest year 2 | - | 60 | - | 10 |
| | - | 660 | - | 110 |
| Receipt year 2 | + | 700 | + | 500 |
| | + | 40 | + | 390 |
| Interest year 3 | + | 4 | + | 39 |
| | + | 44 | + | 429 |
| Receipt year 3 | + | 900 | + | 500 |
| **Summary (£ 000s)** | | **1** | | **2** |
| Net Terminal Value (NTV) | | 944 | | 929 |
| NTV = NPV $(1+r)^3$ | | | | |
| NPV = NTV/$(1+r)^3$ | | 709 | | 698 |

Of course, the above data set could be formulated using each project's IRR as their respective borrowing and reinvestment rates (43 per cent for Project 1 and 54 per cent for Project 2). In both cases the bank surplus (NTV) and its discounted equivalent (NPV) would equal zero. And as we know from the original question, IRR maximisation would select Project 2. Perhaps you can confirm this?

But what is the point, if the company actually borrows at a *real world* (rather than *hypothetical break-even*) rate of 10 per cent for each project? It also seems unreasonable to assume that there are any real world reinvestment opportunities yielding 54 per cent, let alone 43 per cent!

**4: A Graphical Analysis**

Both NPV and IRR models employ common simplifying assumptions that you should be familiar with, one of which is that borrowing and lending rates are equal. But note that

- NPV assumes that projects are financed and intermediate net cash inflows are reinvested at the discount rate.
- IRR assumes that finance and reinvestment occur at that rate where the project breaks even and the NPV equals zero (i.e. the project's IRR).

Given the difference between actual discount rates (r) applied to projects and their IRR, you should also appreciate their impact on the timing and size (pattern) of project cash flows.

> To visualise why a particular discount rate applied to different cash patterns determine their PV and hence NPV and IRR, you could refer to a DCF table for $1/ (1+r)_t$. This reveals the effect of discounting £1.00, $1.00, or whatever currency, at increasing interest rates over longer time periods. Now draw a diagonal line from the top left-hand corner to the bottom right-hand corner of the table (where the figures disappear altogether)? Finally, graph the line. Without being too mathematical, can you summarise its characteristics?

Note that your graph is not only *non-linear* but also *increasingly curvi-linear*. If you are in difficulty, think *compound* interest (not *simple* interest) and reverse its logic. DCF is its *mirror image*, which reveals that for a given discount rate, the longer the discount period, the lower the PV. And for a given discount period, the higher the discount rate, the lower the PV. So, increase the discount rate and extend the discount period and the PV of £1.00 (say) evaporates at an increasing rate.

Applied to our Exercise, a graph should be sketched that compares the two projects, with NPV on the *vertical* axis and discount rates on the *horizonta*l axis, to reveal these characteristics

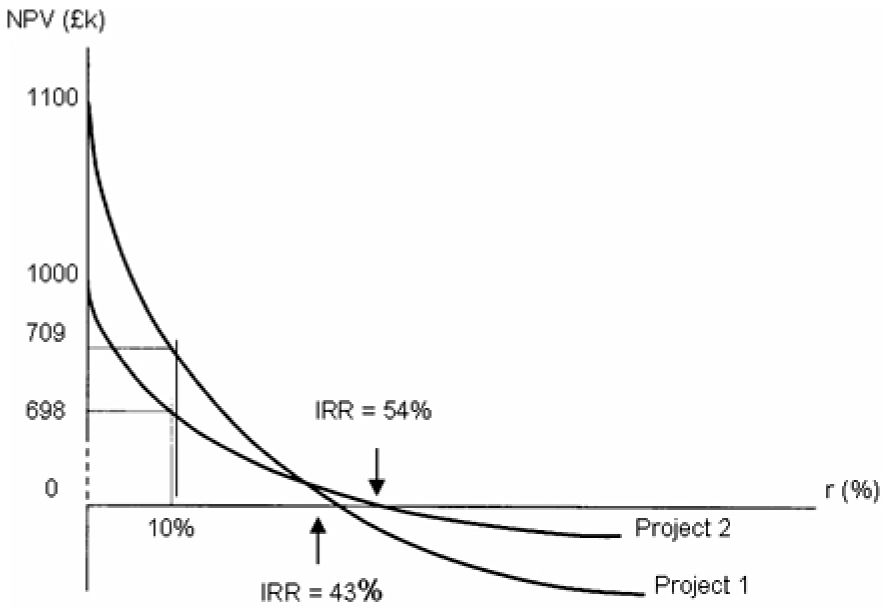


**Figure 3.1:** IRR and NPV Comparisons

Figure 3:1 illustrates that at one extreme (the vertical axis) each project's NPV is maximised when *r* equals zero, since cash flows are not discounted. At the other (the horizontal axis) IRR is maximised because *r* solves for a break-even point (zero NPV) beyond which, both projects under-recover because their NPV is negative.

Using NPV and IRR criteria, the graph also confirms that in *isolation* both projects are acceptable However, if a *choice* must be made between the two, Project 1 maximises NPV, whereas Project 2 maximises IRR. So, why do their NPV curves intersect?

The intersection (crossover point) between the two projects represents an *indifference* point between the two if that was their common discount rate. The NPVs of Project 1 and 2 are the same (any idea of the discount rate and the project NPVs)?. To the left, lower discount rates favour Project 1, whilst to the right; higher rates favour Project 2 leading to its significantly higher IRR.

Refer back to your analysis of PV tables and you should also be able to confirm that:

- NPV (a low discount rate) selects Project 1 because it delivers more money, but later.
- IRR (a high discount rate) selects Project 2 because it delivers less money, but earlier.
- Wealth maximisation equals NPV maximisation (in *absolute* in cash terms) but not necessarily IRR maximisation (a *relative* overall percentage). So, Project 1 is accepted.

Finally, irrespective of the time value of money, if you are still confused about the difference between maximising wealth in *absolute money terms* or maximising a *percentage rate of return*, ask yourself the following simple question:

Is a 20 percent return on £1 million preferable to a 10 percent return on £20 million?

## Exercise 3.2: Relevant Cash Flows, Taxation and Purchasing Power Risk

To supply a university consortium with e-learning material on DVD over the next three years, the South American Clever Publishing Company (CPC) needs to calculate a contract price.

Management believe that the contract's acceptance will enable CPC to access a new area of profitable investment characterised by future growth. This would also reduce the company's reliance on hard copy texts for its traditional clientele. For these reasons management are willing to divert resources from existing projects to meet production. The company will also relax its normal strict terms of sale. The consortium would pay the contract price in two equal instalments; the first up front but the second only when the CPC contract has run its course.

The following information has been prepared relating to the project:

1. **Inventory**
   At today's prices, component costs are expected to be $150,000 per annum. The contract's importance dictates that the requisite stocks will be acquired prior to each year of production. However, sufficient items are currently held in inventory to cover the first year from an aborted project. They originally cost $100,000 but due to their specialised nature, neither the supplier nor competitors will repurchase them. The only alternative is hazardous waste disposal at a cost of $5,000.

2. **Employee Costs**
   Each year the contract will require 3,000 hours of highly skilled technicians. Current wage rates are $8.00 per hour. Because these skills are in short supply, the company would also lose a profit contribution of $2.00 per hour in Year 1 by diverting personnel from an existing project if the contract is accepted.

3. **Overheads**
   Fixed overheads (excluding depreciation) are estimated to be $50,000 at current prices. Variable overheads are currently allocated to projects at a rate of $60.00 per hour of skilled labour.

4. **Capital Investment**
   Fixed assets and working capital (net of inventory) for the project will cost $2 million immediately. The realisable value of the former will be negligible. Company policy is to depreciate assets on a reducing balance basis. When the contract is fulfilled $50,000 of working capital will be recouped.

5. **Taxation**
   Because the contract is marginal in size and the contract deadline is imminent, a decision has been taken to ignore the net tax effect upon the company's revised portfolio of investments if the contract is accepted. However, it is envisaged that the contract itself will attract a $255,000 government grant at the time of initial capital expenditure.

6. **Anticipated Price Level Changes**
   The rate of inflation is expected to increase at an annual compound rate of 15 per cent. Employee costs and overheads will track this figure but component costs will increase at an annual compound rateof 20 per cent.

**Required:**

Assuming that CPC employ a discount rate for new projects based upon an annual cost of capital of 4.5 per cent in *real* terms:

1. Calculate the Clever Company's minimum contract price.
2. Explain your figures.
3. Comment on other factors not reflected in your calculations which might affect the price.

**An Indicative Outline Solution**

**1: The Calculations**

The minimum price at which the Clever Company should implement the project is that which produces a zero NPV. But because our analysis involves price level changes, we must initially ascertain the *Fisher effect* upon the real discount rate explained in Chapter Three (page 46) of the *SFM* text. To the nearest percentage point, this *money* rate (m) is given by:

(5) $m = (1 + r)(1 + i) - 1$
$= (1.045)(1.15) - 1 = 20\%$

Next, the contract's *real* current cash flows must be inflated to *money* cash flows, prior to discounting at the 20 per cent *money* rate.

Using the opportunity cost concept, let us tabulate the contract's *relevant* current cash flows ($000s) attached to their appropriate price level adjustments (in brackets):

| **Year** | **0** | **1** | **2** | **3** |
|---|---|---|---|---|
| **Cashflows** | | | | |
| Capital Investment | -2,000 | | | +50 |
| Capital Allowance | +255 | | | |
| Materials | +5 | -150(1.2) | $-150\ (1.2)^2$ | |
| Labour | | -24 (1.15) | $-24\ (1.15)^2$ | $-24\ (1.15)^3$ |
| Contribution Foregone | | -6 (1.15) | | |
| Variable Overheads | | -180 (1.15) | $-180\ (1.15)^2$ | $-180\ (1.15)^3$ |

**Relevant Real Cash Flows and Price Level Adjustments**

Leaving you to determine the contract's relevant money cash flows, you should now be able to confirm that the application of the money discount rate to the company's net money cash outflows produces the following PV calculations. Using software, a calculator, or DCF tables:

| Year | 0 | 1 | 2 | 3 | PV |
|---|---|---|---|---|---|
| Net Outflows | 1,740.00 | 421.50 | 485.79 | 260.20 | |
| DCF (20%) | 1,740.00 | 351.25 | 337.35 | 150.58 | 2,579.18 |

**PV Calculations ($000s)**

Thus, the minimum contract PV under the conditions stated is $2,579,180. However, remember that the university consortium will only pay this price in two *equal* instalments (Year 0 and Year 3). If the CPC is to break even, we must divide the total payment as follows;

Let C represent the amount of each instalment and the money cost of capital equal 20 percent. Algebraically, the two amounts are represented by the following PV equation ($000s):

$$\$2{,}579.18 = C + \frac{C}{(1.2)^3}$$

Rearranging terms and simplifying, we find that:

$$\frac{\$2{,}579.18}{1.578} = C = \$1{,}634.46$$

And because there is only *one unknown* in the equation, solving for C we can confirm that the minimum contract price of $2,579,180 can be paid in two equal instalments of $1,634,460 now and $1,634,460 in three years time without compromising the integrity of CPC's investment strategy.

**2: An Explanation**

Our contract price calculation is based on the following *relevant* money cash flows discounted at the appropriate *money* cost of capital.

(i) Inventory

There is sufficient stock to maintain first year production. However, its original purchase price is *irrelevant* to our appraisal. It is a *sunk* cost because the only alternative is disposal for $5,000 only avoidable if the contract were accepted. We therefore record this figure as an *opportunity benefit*. At the beginning of Year 2 and Year 3 components have to be purchased at their prevailing prices of $180,000 and $216,000 respectively.

(ii) Employee Costs

3,000 hours of skilled labour will be required each year. If we assume that the company's annual pay award based upon the forecast rate of inflation is impending, the hourly wage rates over three years would be \$9.20, \$10.58 and \$12.17 respectively.

(iii) Contribution Foregone

Because of a skilled labour shortage, the contract's acceptance would lose CPC a contribution of \$2 per skilled labour hour from another project in the first year. We must therefore include \$2 × 3,000 adjusted for inflation as an implicit contract cost.

(iv) Overheads

If fixed overheads are incurred irrespective of contract acceptance they are irrelevant to the decision. Conversely, variable overheads are an incremental cost. They enter into our analysis based on a cost of \$60.00 per hour of skilled labour at \$180,000 adjusted for inflation over each of the three years in accordance with the company's pay policy.

(v) Capital Investment

Depreciation is a *non-cash* expense. Except to the extent that it may act as a tax shield it is therefore irrelevant to our decision. You will recall that since PV analyses are designed to recoup the cost of an investment, depreciation is already incorporated into discounting \$2 million at Year 0 with a zero value for fixed assets at Year 3.

In contrast, that proportion of the $2 million investment represented by working capital is a cash outflow, which will be released for use elsewhere in the company once the contract has run its course. Assuming that $50,000 is the actual amount still tied up at the end of Year 3, we must show this amount as a cash inflow in our calculations.

(vi) Taxation

Because the project is marginal CPC ignored the net tax effect on the overall revision to its investment portfolio. However, we can incorporate the government grant of $255,000 as a cost saving, providing the company proceeds with the project.

**3: Other Factors**

Diversification based on a core technology that uses existing resource elements is a sound business strategy. In this case it should provide new experience in a new sector ripe for exploitation at little risk (the project is marginal).

However, the contract costs (and price) benefit from a project that the company has already aborted. This may indicate a strategic forecasting weakness on the part of management. The lost contribution from diverting resources from any existing project may also entail future loss of goodwill from the company's existing clientele upon which it still depends.

Although the project is marginal, we must also consider whether the company will miss out on more traditional profitable opportunities over the next three years. However, we could argue that if further e-learning contracts follow, their returns will eventually outweigh the risk.

**4: A Conceptual Review**

Our contract appraisal assumes that the data is correct and that net money cash flows can be discounted at a 20 percent money cost of capital. It is based on the following *certainty* assumptions that underpin all our previous PV analyses.

- The costs of investments are known.
- An investment's life is known and will not change.
- Relevant future cash flows are known.
- Price level changes are pre-determined.
- Discount rates based on money (market) rates of interest can be defined and will not change.
- Borrowing and reinvestment rates equal the discount rates.
- The firm can access the capital market at the market rate of interest if internal funds are insufficient to finance the project, or interim net cash flows are available for reinvestment.

Uncertainty about any one of these assumptions is likely to invalidate our investment decision and compromise shareholders' wealth.

In the contract calculations, it may increase the minimum contract price far beyond $2,579,180.

## Summary and Conclusions

A project's NPV is equivalent to the PV of the net cash surplus at the end of its life (NTV). We observed that this should equal the project's *relevant* cash flows discounted at appropriate *price-adjusted* opportunity cost of capital rates, using prevailing rates of interest or the company's desired rate of return. To maximise wealth, management should then select projects with the highest NPV to produce the highest lifetime cash surplus (NTV).

It is also worth repeating from Exercise 3.1 that the NPV approach to investment appraisal based on *actual* DCF cost or return cut-off rates should be more realistic than an IRR.

The IRR model is an *arithmetic* computation with little economic foundation. It is a *percentage averaging technique* that merely establishes a project's *overall break-even* discount rate where the NPV and NTV (the cash surplus) equal zero. Moreover, IRR may rank projects in a different order to NPV. This arises because of different cash flow patterns and the disparity between a project's IRR and a company's opportunity capital cost (or return) each of which determines the borrowing and reinvestment assumptions of the respective models.

Of course, the assumptions of NPV analyses presented so far ignore the uncertain world inhabited by management, each of which may invalidate the model's conclusions.

So, as a companion to the *SFM* text, let us develop the NPV capital budgeting model in Chapter 4 by illustrating a number of formal techniques that can reduce, if not eliminate, the risk associated with strategic investment appraisal.

# 4 The Treatment of Uncertainty

## Introduction

For simplicity, our previous analyses of investment decisions assumed the future to be certain. But what about the real world of uncertainty, where cash flows cannot be specified in advance? How do management maximise their strategic NPV wealth objectives?

In Chapter Four of your companion text (*SFM*) we evaluated risky projects where more than one set of cash flows are possible, based on two statistical parameters, namely the *mean* and *standard deviation* of their distribution. But do you understand them?

One lesson from the recent financial meltdown is that irrespective of whether you are sitting an examination, or dealing with multi-national sub-prime mortgages on Wall Street, a good memory for formulae, access to a simple scientific calculator, or the most sophisticated software, is no substitute for understanding what you are doing and its consequences.

Using mean-variance analysis as a springboard, the following exercises therefore emphasise: why you should always be able to explain what you are calculating, know what the results mean, are critically aware of their limitations and how the analysis may be improved. Real financial decisions should always consider "what is" and "what should be".

**Exercise 4.1: Mean-Variance Analyses**

| Project | Mean NPV | Standard Deviation of NPV |
|---|---|---|
| | € (000s) | € (000s) |
| A | 39 | 27 |
| B | 27 | 27 |
| C | 39 | 33 |
| D | 45 | 36 |

The above table summarises statistical data for a series of *mutually exclusive* projects under review by the Euro Song Company (ESCo).

**Required:**

1. Prior to analysing the data set, summarise in *your own words*:
   - The formal statistical assumptions that underpin mean-variance analysis.
   - The definition of a project's mean, the variance and purpose of the standard deviation.
2. Reformulate the data set to select and critically evaluate the most efficient project based on the various mean-variance criteria explained in Chapter Four of the *SFM* companion text.
3. Explain the limitations of your findings with reference to the *risk-return paradox*.

**An Indicative Outline Solution**

**1: Summary**

**- The Formal Assumptions**

For the purpose of risk analysis, most financial theorists and analysts accept the statistical assumptions of *classical* probability theory, whereby:

- Cash flows are *random* variables that are *normally* distributed around their *mean* value.
- *Normal* variables display a *symmetrical* frequency distribution, which conforms to a bell shaped curve (see below) based on the *Law of Large Numbers*.
- The Law's *Central Limit Theorem* states that as a sample of independent, identically distributed (*IID*) random numbers (i.e. cash flow variables) approaches infinity, its probability density function will conform to the normal distribution. If variables are normally distributed, a finite, statistical measure of their dispersion can be measured by their *variance*.

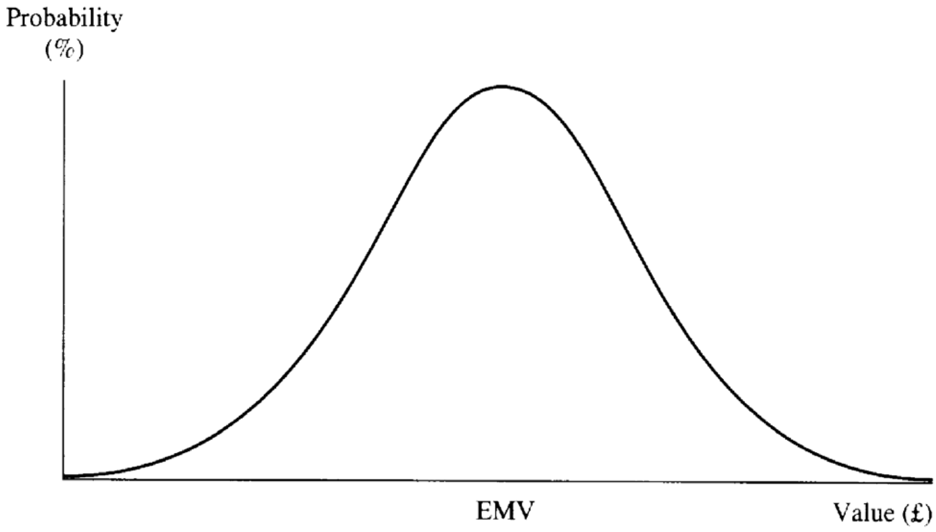


**Figure 4.1:** A Normal Distribution (£Cash Flows)

**- Definitions**

The *mean* (average) return from a project is a measure of *location* given by the weighted addition of each return. Each weight represents the probability of occurrence, subject to the proviso that project's returns are random variables and the sum of probabilities equals one.

The *variance* of a project's returns (risk) is a measure of *dispersion* equal to the weighted addition of the squared deviations of each return from the mean return. Again, each weight is represented by the probability of occurrence.

The *standard deviation* of a project is simply the *square root* of the variance.

So, what does the standard deviation contribute to our analysis of risk?

Because the distribution of normal returns is *symmetrical*, having calculated the deviation of each return from the project mean, we cannot simply weight the deviations by their probabilities to arrive at a mean deviation as a measure of dispersion. Unless the investment is *riskless*, some deviations will be positive, others negative, but *collectively* the mean deviation would still equal zero. We also know that the sum of all probabilities always equals one, so the mean deviation remains zero.

So, if we first square the deviations, we eliminate the minus signs and derive the *variance*. But in relation to the original mean of the distribution, we now have a *scale* problem.

The increased scale, through squaring, is remedied by calculating the *square root of the variance*. This equals the *standard deviation*, which is a measure of dispersion expressed in the *same units* as the mean of the distribution.

Thus, management have an NPV risk-return model where both parameters are in the same monetary denomination (€ in our current example). Thus, if a choice must be made between alternatives, the firm's wealth maximisation objective can be summarised as follows:

> Maximise project returns at minimum risk by comparing their expected net present value (ENPV) with their standard deviation (σ NPV).

**2: Efficient Project Selection**

As a summary measure of project risk based on the dispersion of cash flows around their mean, the interpretation of the standard deviation seems obvious: the higher its value, the greater the risk and *vice versa*.

However, projects that produce *either* the highest mean return (ENPV) *or* the lowest dispersion of returns (σNPV) are not necessarily the least risky. The *total* risk of a project must be assessed by reference to *both* parameters and compared with alternative investments.

To evaluate projects that are either *mutually exclusive* or subject to *capital rationing*, the depth of variability around the mean must be incorporated into our analysis. We can either maximise the expected return for a given level of risk, or minimise risk for a given expected return

MAX: ENPV, given σNPV or MIN: σNPV, given ENPV

Ideally, we should also maximise ENPV and minimise σ NPV using a *risk-free* discount rate to avoid double counting. So, let us refer to the data set and analyse its risk- return profile.

Project A has a higher expected rate of return than project B but the standard deviation is the same, so project A is preferable. Project C has the same mean as project A but has a larger standard deviation, so it is inferior. The most efficient choice between A, B and C is therefore project A.

However, we encounter a problem when comparing projects A and D, since D has a higher mean *and* a higher standard deviation. So, which one of these projects should ESCo accept?

**- The *z* statistic**

You will recall from the *SFM* text that if cash flows ($C_i$) are normally distributed, we can use the statistical table for the *area under the standard normal curve* to establish the probability that any value will lie within a given number of standard deviations away from their mean (EMV) by calculating the *z* statistic. The mechanistic procedure is as follows:

Calculate how many standard deviations away from the mean is the requisite value. This is given by the *z statistic,* which measures the actual deviation from the mean divided by the standard deviation. So, using Equation (5) from the *SFM* text:

$$(5)\ z = C_i - EMV / \sigma (C_i)$$

Next, consult the table to establish the area under the normal curve between the right *or* left of *z* (plus or minus) by finding the absolute value of *z* to two decimal places.

For example, the area one standard deviation above the mean is found by cross-referencing the first two significant figures in the left hand column (1.0) with the third figure across the top (0).Therefore, the probability of a value lying between the mean and one standard deviation above the mean is 0.3413, which equals 34.13 per cent.

Since a normal distribution is symmetrical, 2*z* represents the probability of a variable deviating above *or* below the mean. Therefore, the probability of a value +σ, or -σ, away from the mean corresponds to 68.26 percent of the total area under the normal curve, i.e. twice 34.13 percent.

As a measure of risk, the standard deviation has a further convenient property in relation to the normal curve. Assuming normality, we have estimated the percentage probability that any variable will lie within a given number of standard deviations from the mean of its distribution by calculating the *z* statistic.

Reversing this logic, from a table of *z* statistics we can observe that any normal distribution of random variables about their mean measured by the standard deviation will conform to prescribed *confidence limits*, which we can express as a percentage probability.

For example, the percentage probability of any cash flow ($C_i$) lying one, two or three standard deviations above or below the EMV of its distribution is given by:

EMV ± nσ (where n equals the number of standard deviations)

| | | |
|---|---|---|
| 2 x 0.3413 | for -σ to +σ | = 68.26% |
| 2 x 0.4772 | for -2σ to +2σ | = 95.44% |
| 2 x 0.4987 | for -3σ to +3σ | = 99.74% |

(Perhaps you can confirm these figures by reference to a *z* table?)

Returning to our data set, let us assume that the management of ESCo wish to choose between projects A and D using an approximate confidence limit of 68 percent. The basis for their accept reject decisions can be summarised as follows: (€000s)

| Probability | | ENPV ± nσ (where n equals one) | |
|---|---|---|---|
| 2 × 0.3413 = 68.26% | for | -σ to | +σ |
| Project A: ENPV | | 39-27 =12 | 39+27 = 66 |
| Project D: ENPV | | 45-36 = 9 | 45+36 = 81 |

Unfortunately, the company still cannot conclude which project is less risky. Explained simply, should it opt for project A with the likelihood of €12,000 (compared to only €9,000 from project D) or project D with an equal likelihood of €81,000 (compared to €66,000 from project A)?

**- The Coefficient of Variation**

To resolve the problem, one solution (or so it is argued) is to measure the *depth* of variability from the mean using a *relative* measure of risk (rather than the standard deviation alone, which is an *absolute* measure). Using Equation (6) from the *SFM* text, we could therefore apply the *coefficient of variation* to our project data set (€000s) as follows:

$$(6)\ \text{Coeff.Var.} = (\sigma\ \text{NPV}) / (\text{ENPV})$$

Project: A 27/39 = 0.69; B 27/27 = 1.00; C 33/39 = 0.85; D 36/45 = 0.80.

These figures now confirm that projects B and C are more risky than A and D. Moreover, D is apparently more risky than A because it involves €0.80 of risk (standard deviation of NPV) for every €1.00 of ENPV, whereas project A only involves €0.69 for every €1.00 of ENPV. So, should the management of ESCo now select project A?

**- The Profitability Index**

Unfortunately, we still don't know. The coefficient of variation (like the IRR under certainty) ignores the *size* of projects, thereby assuming that risk attitudes are *constant*. Add zeros to the previous project data and note that the coefficients would still remain the same. Yet, intuitively, we all know that investors (including management) become increasingly risk averse as the stakes rise. Explained simply, is a low coefficient on a high capital investment better than a high coefficient on a low capital investment or *vice versa*?

To overcome the problem, an alternative solution is for management to predetermine a desired *minimum* ENPV for investment ($I_0$) expressed as a *profitability index* with which they feel comfortable. This *benchmark* can then be compared to *expected* indices for proposed investments, which also incorporate *confidence limits* (as defined earlier) to reflect *subjective* managerial risk attitudes. So, the company's objective function for project selection using Equation (7) from the *SFM* text becomes:

$$(7)\ \text{MAX: } (\text{ENPV} - n\sigma\text{NPV}) / I_0 \geq \text{MIN: NPV} / I_0$$

Assume that ESCo apply a benchmark [MIN:NPV / $I_0$] = €0.12 to satisfy stakeholders. Use the initial data to derive the left-hand side of Equation (7) one standard deviation from the mean for projects A and D, assuming that they cost €100k and €75k, respectively. Now use the whole equation to compare their acceptability to management.

Recall that mean-variance analysis alone (or the *z* statistic and confidence limits) could not discriminate between project A or D. Using the coefficient of variation, Project A seemed preferable to D. Note now, however, using the expected profitability index with the same confidence interval (68.26% probability) that both projects are equally acceptable (€ 000s).

| Project | (ENPV – σNPV) / $I_0$ | ≥ | MIN: £NPV / $I_0$ | Decision |
|---|---|---|---|---|
| A | 39 – 27 /100 = 0.12 | = | 0.12 | Accept |
| D | 45 – 36 / 75 = 0.12 | = | 0.12 | Accept |

But is this true?

**- The Risk-Return Paradox**

From your reading you should be aware that modern financial theory defines investors (including management) as rational, risk-averse investors who seek maximum returns at minimum risk. But throughout our example, we have a statistical-behavioural *paradox* based on the *symmetric normality* of returns and their *depth* of variability around the mean, however we define it.

ESCo still cannot conclude which project is less risky. Explained simply, one standard deviation from the mean, should it opt for project A with the likelihood of €12,000 (compared to only €9,000 from project D) or project D with an equal likelihood of €81,000 (compared to €66,000 from project A)?

Whilst project A maximises *downside* returns there is also an equal probability that project D maximises *upside* returns. So, is project A less risky than project D?

Below the mean, risk aversion would select the former, (why?). Above the mean, project D is clearly more attractive, (to whom?). Presumably, rational, risk-averse investors would say "yes" to project A. But those prepared to gamble would opt for project D?

The *risk-return paradox* cannot be resolved by formal, statistical analyses of the mean, variance or standard deviation, confidence limits, *z* statistics, and coefficients of variation, or profitability indices.

We also need to know the *behavioural* attitudes of decision-makers (in our example, the corporate management of ESCo) towards risk (aversion, indifference, or preference). And for this you must refer back to Chapter Four of the *SFM* text for the concept of *investor utility* and the application of *certainty equivalent* analysis within the context of investment appraisal.

## Exercise 4.2: Decision Trees and Risk Analyses

Our previous exercise considered statistical techniques for selecting investments based on their *predetermined* pattern of probabilistic cash flows However, companies are sometimes faced with more complex *sequential* decisions where:

> Management need to make a strategic choice between alternative courses of action with the possibility of future alternative courses of action occurring dependant upon their previous choices.

In Chapter Four of the *SFM* text, we therefore mentioned a diagrammatic technique termed "decision tree" analysis to clarify this problem. The diagram begins with the investment decision (trunk) which is then channelled through alternative strategies (branches) arising from subsequent managerial decisions (*control* factors) or pure chance. As each branch divides, (*nodal* points) monetary values and *conditional* probabilities are attached until all possible outcomes are exhausted. Each *node* represents a *decision point* that departs from previous decisions, stretching back to the initial investment. Moving up the tree, the branch structure therefore reveals eventual possible profits (or losses) in terms of EMV. NPV techniques using mean-variance analyses can then be applied to assess an optimum investment decision.

So, let us illustrate the technique using the following information.

The Chilli Pepper Group (CPG) needs a new productive process, the cost of which is either £2 million or £3 million depending on future demand. The following forecast data is available.

| | **£2m Project** | | | **£3m Project** | |
|---|---|---|---|---|---|
| **Probability** | **Years** | **Annual cash flow (£m)** | **Probability** | **Years** | **Annual cash flow (£m)** |
| 0.4 | 1–4 | 0.60 | 0.3 | 1–4 | 1.00 |
| | 5–10 | 0.50 | | 5–10 | 0.70 |
| 0.4 | 1–4 | 0.60 | 0.5 | 1–4 | 0.80 |
| | 5–10 | 0.20 | | 5–10 | 0.40 |
| 0.2 | 1–10 | 0.20 | 0.2 | 1–10 | 0.10 |
| $\Sigma P_i = 1.0$ | | | $\Sigma P_i = 1.0$ | | |
| Cost of Capital 10% | | | Cost of Capital 10% | | |

**Required:**

1. Prior to analysing the data set, refer to appropriate DCF tables and summarise the factors necessary for your analysis.
2. Use the data set and your DCF factors to determine the probabilistic cash flows for both investment opportunities (£2m and £3m).
3. Analyse all the above information in the form of decision trees.
4. Comment on the statistical validity of your findings.

**An Indicative Outline Solution**

1. **The Present Value of £1.00 received annually for the requisite number of years.**

| Years | 10% DCF Factor |
|---|---|
| 1–4 | 3.17 |
| 5–10 | 2.98 |
| 1–10 | 6.15 |

2. **The Present Value of Probabilistic Cash Flows Discounted at 10 per cent (£m)**

| | | £2m | Project | £3m | Project |
|---|---|---|---|---|---|
| **Years** | **PV Factor** | **Cash Flows** | **PV** | **Cash Flows** | **PV** |
| 1–4 | 3.17 | 0.60 | 1.90 | 1.00 | 3.17 |
| 5–10 | 2.98 | 0.50 | 1.50 | 0.70 | 2.09 |
| 1–4 | 3.17 | 0.60 | 1.90 | 0.80 | 2.54 |
| 5–10 | 2.98 | 0.20 | 0.60 | 0.40 | 1.19 |
| 1–10 | 6.15 | 0.20 | 1.23 | 0.10 | 0.62 |

3. **The Decision Trees (£m)**

All the previous information for either project can be graphically reformulated as a decision tree summarised below. Each diagram (£m) begins with the initial decision (£2m or £3m), moving through the branches associated with their alternative strategic pay-offs. For convenience, their respective ENPV at 10 per cent is also shown.

**£ 2 Million Investment**

| Project | Probability | Years | Cash Flow | PV Cash Flow | Expected PV |
|---|---|---|---|---|---|
| 2 | 0.4 | 1-4 | 0.6 | 1.90 | 1.36 |
| | | 5-10 | 0.5 | 1.50 | |
| | 0.4 | 1-4 | 0.6 | 1.90 | 1.00 |
| | | 5-10 | 0.2 | 0.60 | |
| | 0.2 | 1-10 | 0.2 | 1.23 | 0.25 |
| | $\Sigma P_i$ 1.0 | | | EPV | 2.61 |
| | | | | Cost | 2.00 |
| | | | | ENPV | 0.61 |

**£3 Million Investment**

| Project | Probability | Years | Cash Flow | PV Cash Flow | Expected PV |
|---|---|---|---|---|---|
| 3 | 0.3 | 1-4 | 1.0 | 3.17 | 1.58 |
| | | 5-10 | 0.7 | 2.09 | |
| | 0.5 | 1-4 | 0.8 | 2.54 | 1.87 |
| | | 5-10 | 0.4 | 1.19 | |
| | 0.2 | 1-10 | 0.1 | 0.62 | 0.12 |
| | $\Sigma P_i$ 1.0 | | | EPV | 3.57 |
| | | | | Cost | 3.00 |
| | | | | ENPV | 0.57 |

The two decision trees reveal that the ENPV of the £2 million investment is marginally superior to that for £3 million. So, presumably, the incremental investment of £1 million is not worthwhile? However, look closely at the data and you will see a larger range of possible outcomes for the larger investment (i.e. a greater chance of lower cash flows *but also* a greater chance of higher cash flows). So, is the smaller investment really preferable?

4. **Statistical Commentary**
The first point to note is that if the worst and best *states of the world* materialise, the £2 million investment *minimises* losses whilst £3 million *maximises* profits.

| Worst Case Scenario | £2 million Project | £3 million Project |
|---|---|---|
| Cash flow | 1.23 | 0.62 |
| Investment | (2.00) | (3.00) |
| NPV | **(0.77)** | (2.38) |

| Best Case Scenario | £2 million Project | £3 million Project |
|---|---|---|
| Cash flow | 3.40 | 5.26 |
| Investment | (2.00) | (3.00) |
| NPV | 1.40 | **2.36** |

Thus, we might conclude that if CPC wish to take a chance it could opt for the larger investment. However, any risk assessment should be guided by the investment's size relative to the scale of the company's other operations. If £3 million represents a *marginal* investment in a diverse, multi-project firm, then management need not worry unduly. But if CPC is small with a narrow investment portfolio, the failure of this one project could be catastrophic.

So, let us focus on the *downside* risk for each project using mean-variance analysis, given:

EMV = ENPV (@ £2m) = 0.61 > ENPV (@ £3m) = 0.57

Each project's standard deviation is calculated using the PV of cash flows for their branches For example, the $C_i$ of 3.4 in the first cell below equals 1.9 plus 1.5 (£ million) used earlier.

| $C_i$ | $P_i$ | $C_i P_i$ | $(C_i – EMV)^2$ | $P_i$ | $(C_i – EMV)^2 P_i$ |
|---|---|---|---|---|---|
| 3.40 | 0.4 | 1.36 | 0.62 | 0.4 | 0.248 |
| 2.50 | 0.4 | 1.00 | 0.012 | 0.4 | 0.005 |
| 1.23 | 0.2 | 0.25 | 1.90 | 0.2 | 0.380 |
| S $P_i$ | 1.0 | | | 1.0 | |
| Expected Monetary Value (EPV) 2.61 | | | Variance (VAR = $\sigma^2$ ) = 0.633 | | |
| ENPV = EPV – $I_0$ = 2.61 – 2.00 = 0.61 | | | S.D. ($\sqrt{VAR} = \sigma$) = 0.796 | | |

**Mean-Variance Analysis at £2 Million**

| $C_i$ | $P_i$ | $C_i P_i$ | $(C_i – EMV)^2$ | $P_i$ | $(C_i – EMV)^2 P_i$ |
|---|---|---|---|---|---|
| 5.26 | 0.3 | 1.58 | 2.86 | 0.3 | 0.858 |
| 3.73 | 0.5 | 1.87 | 0.03 | 0.5 | 0.015 |
| 0.62 | 0.2 | 0.12 | 8.70 | 0.2 | 1.740 |
| S $P_i$ | 1.0 | | | 1.0 | |
| Expected Monetary Value (EPV) 3.57 | | | Variance (VAR = $\sigma^2$ ) = 2.613 | | |
| ENPV = EPV – $I_0$ = 3.57 – 3.00 = 0.57 | | | S.D. ($\sqrt{VAR} = \sigma$) = 1.615 | | |

**Mean-Variance Analysis at £3 Million**

You might care to confirm that the £2 million investment minimises downside returns at all confidence levels.

## Summary and Conclusions

Our first Exercise dealt with risky projects where more than one set of cash flows are possible, based on two classical statistical parameters, namely the *mean* and *standard deviation* of their distribution. However, despite the increasing sophistication of our analyses, none of the models specified investors risk attitudes (for example managerial reaction to confidence intervals). We therefore suggested reference to an even more sophisticated approach to investment appraisal, covered in Chapter Four of your companion text (*SFM*) namely:

**The PV maximisation of the expected utility of cash equivalents**

However, this model too, is problematical. Its validity still depends upon how basic financial data feeds into complex ENPV calculations. And this is where our second Exercise fits in.

Decision trees (like sensitivity analysis and computer simulation also covered in the *SFM* text) are not selection criteria, but an aid to judgement. They do not provide *new* information. However, they do clarify *crucial* information using sequential decision points and their probabilistic outcomes in *simple* financial terms. Perhaps strategic management ought to return to this technique and adopt a more "hands on" approach to investment appraisal, rather than rely on "hands off" computer programs, which use incomprehensible models that precipitated the ongoing 2008 global and financial and economic meltdown, so often referred to throughout our study.

# **Part Three**
# The Finance Decision

# 5 Equity Valuation the Cost of Capital

## Introduction

Having explained how ENPV investment models can maximise shareholder wealth, we need to consider how management actually finance investments, since their cost of capital determines project discount rates and hence corporate value. Part Three of the *SFM* text reveals how funds can be raised from a variety of sources at different costs with important implications for a company's *overall* discount rate and shareholder wealth. However, even the derivation of a *single* discount rate in *all-equity* firms poses problems. To maximise wealth, management need to know their shareholders' desired rate of return and then only accept projects with a positive ENPV discounted at this rate. But this not only presupposes a share valuation model that determines the current return on equity but also the nature of the return. Is it a dividend or earnings stream?

Chapter Five of our companion text touched on this problem in the Review Activity. The following exercises examine its complexity in more detail. Each question begins with an exposition of the theories required for its solution. And because of their complexity, we shall develop the data throughout both exercises in a "case study" format (so you can retrace your steps back from the second exercise to the first if necessary). To tackle the sequence of questions throughout this chapter you also need to refer to the *SFM* text, other readings you are familiar with, plus your knowledge of share price listings in the financial press.

## Exercise 5.1: Dividend Valuation and Capital Cost

You will recall that Chapter Five of *SFM* defines a company's current *ex div* share price ($P_o$) in a *variety* of ways using the present value model. Each price corresponds to a dividend or earnings stream ($D_t$ or $E_t$) under growth (g) or non-growth conditions, discounted at an appropriate cost of equity ($K_e$) i.e. shareholder return within a specified time continuum.

For example, if shares are held in *perpetuity* and the latest reported dividend per share remains *constant* indefinitely (i.e. g = zero) the current *ex div* price can be expressed using $K_e$ as the shareholders' capitalisation rate for a *perpetual annuity*.

$$P_0 = D_1 / K_e$$

Likewise, a corresponding earnings valuation based on earnings per share (EPS) is given by:

$$P_0 = E_1 / K_e$$

However, rearrange either equation to define the shareholders' return ($K_e$) as a managerial cut-off for investment (project discount rate) and we encounter a fundamental problem.

Assume funds are retained for reinvestment, i.e. dividends are lower than earnings (which characterises the financial policy of most real world companies). Because the *same* share cannot trade at *different* prices at the *same* time, the equity capitalisation rate (discount rate) must *differ* in the two equations. Summarised mathematically, if:

$$D_t < E_t \qquad \text{but } P_0 = D_t / K_e = P_0 = E_t / K_e \text{ then} \qquad K_e = D_t / P_0 < K_e = E_t / P_0$$

Moreover, if $P_0$ is common to both value equations, then not only must the equity yield for dividends and earnings ($K_e$) differ, but a *unique* relationship must also exist between the two.

**The Theoretical Background**

Throughout the 1950s and 1960s, Myron J. Gordon (referenced in *SFM*) formalised the relationship between dividend-reinvestment policies, their associated returns and current share prices under conditions of *certainty* and *uncertainty*. Using a *constant growth* formula:

> The Gordon dividend growth valuation model determines the current ex-div price of a share by capitalising next year's dividend at the amount by which the shareholders' desired rate of return exceeds the constant annual growth rate of dividends.

Using Gordon's notation, where $K_e$ is the equity capitalisation rate; $E_1$ equals next year's post-tax earnings; b is the proportion retained; $[E_1 (1\text{-}b)]$ is next year's dividend; r is the return on reinvestment and rb equals the constant annual growth in dividends, we can define:

$$P_0 = [E_1(1\text{-}b)] / K_e - rb$$

In most Finance texts the equation's notation is simplified as follows, with $D_1$ and g representing Gordon's dividend term and growth rate respectively:

$$P_0 = D_1 / K_e - g$$

Subject to the non-negativity constraint that $K_e > rb = g$ (for share price to be *finite*) we can also rearrange the terms of the Gordon *valuation* model and solve for Ke to produce an *investment* model.

$$K_e = [\{E_1(1\text{-}b)\} / P_0] + rb = (D_1 / P_0) + g$$

> According to Gordon, the managerial cut-off (project discount) rate for new investment is defined by the shareholders' total return, which equals a dividend expectation divided by current share price, plus a premium for growth (capital gain).

Gordon then analysed the behaviour of his models, assuming a perfect world of *certainty* and came to the same conclusions as Irving Fisher thirty years earlier (see Chapter One of *SFM*). According to Fisher's *Separation Theorem*, price movements and returns relate to profitable investment policy and not dividend policy. Specifically:

(i) Shareholder wealth (price and return) will stay the same if r equals $K_e$
(ii) Shareholder wealth (price and return) will increase if r is greater than $K_e$
(iii) Shareholder wealth (price and return) will decrease if r is lower than $K_e$

Thus, dividend policy is a *residual* managerial decision only made once a company's profitable reinvestment opportunities are exhausted.

**A Practical Illustration – Certainty**

To gauge the impact of corporate reinvestment policy on share price and returns using the Gordon growth model under conditions of *certainty* consider the following data.

Because of recession, the share price of Jovi plc tumbled from £2.00 to 75 pence throughout 2009 and market capitalisation fell from £20 million to £7.5 million. EPS and dividend cover also halved, falling from 10 pence to 5 pence and from two to one, respectively. With economic revival, however, Jovi intends to declare a 10 pence dividend per share (covered once) equivalent to a dividend yield of 2.5 per cent.

**Required:**

1. Calculate the new equilibrium price for Jovi's shares based on its dividend intentions.
2. Calculate the new *equilibrium* price if Jovi retained 50 per cent of its annual earnings
3. Comment on your results with regard to shareholder returns and the managerial cut-off rate.

**An Indicative Outline Solution**

1. **The Equilibrium Price (zero growth)**
   Without further injections of capital, a 10 pence dividend covered once not only implies an EPS of 10 pence but an intention to pursue a policy of *full distribution* with *zero* growth. If shareholders are satisfied with a 2.5 per cent yield on this investment, we can define their current share price by using the capitalisation of a *perpetual annuity*.

$$P_0 = E_1 / K_e = D_1 / K_e = 10 \text{ pence} / 0.025 = £4.00$$

2. **The Equilibrium Price (growth)**
   With the same EPS forecast of 10 pence but 50 percent reinvested in perpetuity, new project returns should *at least equal* the original equity capitalisation rate of 2.5 per cent (Fisher's Theorem). So, using this figure for the annual reinvestment rate we can determine an annual growth rate to incorporate into the Gordon valuation model as follows:

$$P_0 = [E_1(1-b)] / K_e - rb = P_0 = D_1 / K_e - g = 5 \text{ pence} / 0.025 - 0.0125 = £4.00$$

3. **Commentary**
   Despite changing mathematical formulae from the capitalisation of a perpetual annuity to a model that accommodates retentions and reinvestment (growth) share price remains the same. Moreover, reformulate the growth equation solving for $K_e$ and it is still equivalent to the original dividend yield; but why?

$$K_e = (D_1 / P_0) + g = (5 \text{ pence} / £4.00) + 0.0125 = 2.5\%$$

According to Gordon, movements in share price relate to the profitability of corporate investment opportunities and not alterations in dividend policy. So, if the company's rate of return on reinvestment (r) equals the shareholders' original capitalisation rate, share price and $K_e$ remain the same (in Jovi's case, 2.5 per cent). Thus, it also follows logically that:

(i) Shareholder wealth (price and return) will *increase* if r is greater than the original $K_e$
(ii) Shareholder wealth (price and return) will *decrease* if r is lower than the original $K_e$

Given $P_0$ = £4.00, $K_e$ = 2.5 per cent and b = 0.5, perhaps you can confirm that if Jovi's reinvestment rate (r) moves from 2.5 per cent *up* to 4.0 per cent or *down* to 1.0 per cent:

$P_0$ moves to £10.00 or £2.50 with corresponding revisions to the cut-off rate ($K_e$) of 3.25 per cent and 1.75 per cent, respectively.

**A Practical Illustration – Uncertainty**

Gordon's initial value-investment model depends on *certainty* assumptions in perfect markets. He begins with a *constant* equity capitalisation rate ($K_e$) *equivalent* to a managerial assessment of a *constant* return (r) on new projects financed by a *constant* retention rate (b).When he changes the variables, they too, remain the same in perpetuity. However, these simplifying assumptions do not invalidate his analysis. Like most financial models they are a *means to an end*. With simple policy prescriptions as *benchmarks*, Gordon moves into the real world by asking "what if the future is *uncertain*"?

According to Gordon, most real-world market participants are still *rational-risk averse* investors who subscribe to a "bird in the hand" philosophy. They prefer *more* dividends *now* rather than later, even if future retentions are more profitable than their current capitalisation rate ($r > K_e$). Consequently, near dividends are valued more highly. Investors discount *current* dividends at a lower rate than *future* dividends ($K_{e1}<K_{e2}<K_{e3}$….) because they expect a higher overall return on equity ($K_{e0}$) from firms that retain a greater proportion of their earnings. The inevitable implication of this *risk- return trade-off* is that share price will fall because equity values are:

- *Positively* related to the dividend payout ratio
- *Inversely* related to dividend cover
- Inversely related to the retention rate
- *Inversely* related to the dividend growth rate.

In a world of *uncertainty* Gordon therefore reverses the logic of his *certainty* argument. He *hypothesises* that dividend policy, rather than investment policy, should motivate management to maximise shareholder wealth. The overall equity capitalisation rate is no longer a *constant* but a function of the *timing and size* of the dividend payout ratio. *Increased* retention rates (delayed dividend payments) result in the most significant *rise* in periodic dividend capitalisation rates and corresponding *fall* in current shares values (or *vice versa*).

*To summarise Gordon's uncertainty hypothesis*, current shareholder returns and managerial cut-off rates are functionally related to the dividend payout ratio, or equivalent retention rate, as follows:

$$K_{e0} = f\,(K_{e1} < K_{e2} < \dots K_{en})$$

Because the greatest periodic inequalities relate to the non-payment of dividends, an *all-equity* firm should *maximise* its dividend payout to *minimise* the equity capitalisation (cut-off) rate and *maximise* share price and corporate wealth.

So, let us focus on the *uncertain* relationships between dividend-investment policies, share price behaviour and managerial discount rates in the presence of retention financed growth.

Consider the following data set for Jovi plc in a world of *uncertainty*. The first line (1) represents a full distribution policy (like our previous example). The second (2) reflects a rational managerial decision to withhold half the dividend (like before). And note, that the company's revised return on reinvestment not only exceeds the company's original capitalisation rate (2.5 per cent) but also the shareholders' upward risk-return revision.

| Forecast EPS | Retention Rate | Dividend Payout | Return on Investment | Growth Rate | Shareholder Return |
|---|---|---|---|---|---|
| $E_1$ | (b) | (1-b) | (r) | rb = g | $K_e$ |
| 1: £0.10 | 0 | 1.0 | - | - | 0.025 |
| 2: £0.10 | 0.5 | 0.5 | 0.075 | 0.0375 | 0.050 |

**Required:**

1. Explain why the basic requirements of the Gordon growth model under conditions of uncertainty are satisfied by the data set.
2. Confirm whether share prices derived from the data set support Gordon's hypothesis.
3. Summarise the conceptual and statistical weakness of on your findings.

**An Indicative Outline Solution**

In Gordon's world of *uncertainty*, share price, equity capitalisation and managerial cut-off rates are a function of dividends-retention policies that are *imperfect* economic substitutes.

**1: The Gordon Model**

Moving from full to partial distribution, our data set *satisfies all the requirements* of the Gordon model. Withholding dividends [$E_1(1–b)=D_1$], to finance new investment not only accords with Fisher's wealth maximisation criterion ($r>K_e$) but also satisfies the mathematical constraint that $K_e > rb = g$. The equity capitalisation rate ($K_e$) also rises with the increased rate of return (r) on retentions (b) i.e. the growth rate (g).

But has share price ($P_0$) fallen, given the reduction in the dividend payout, the increase in growth and $K_e$ and as Gordon predicts?

**2: Gordon's Predictions**

Rational, risk-averse investors may prefer dividends now, rather than later (a "bird in the hand" philosophy that values current consumption more highly than future investment). But using our data set, which *satisfies all the requirements* of the Gordon dividend growth model under conditions of uncertainty, you should have discovered that:

**Despite a change in dividend policy, share price remains the same**

$$P_0 = [E_1(1\text{-}b)] / K_e – rb = P_0 = D_1 / K_e – g = £4.00$$

Of course, the series of variables in the data set were deliberately chosen to ensure that share price remained unchanged. But the important point is that they all satisfy the requirements of the Gordon model, yet contradict his prediction that share price should fall. Moreover, it would be just as easy to produce other data sets, which satisfy his requirement that share price should apparently rise but actually stay the same, (or even fall).

**3: The Gordon Weakness**

The dividend growth model confuses financial policy (*financial risk*) with investment policy (*business risk*). An increase in the dividend payout ratio, without any additional finance, reduces a firm's investment capability and *vice versa*. Consider the basic equation:

$$P_0 = D_1 / K_e - g$$

Change $D_1$, then you change $K_e$ and g.. So, how do you unscramble the differential effects on price ($P_0$) when all the variables on the *right hand side* of the equation are now affected?

Gordon encountered this problem when empirically testing his model, being unable to conclude that dividends determine share price and return. Yet, statisticians among you will recognise the phenomena, termed *multicolinearity*. Change one variable and you change them all because they are all interrelated. No wonder subsequent research, even using sensitivity analysis cannot prove conclusively that dividend policy determines share price.

## Exercise 5.2: Dividend Irrelevancy and Capital Cost

The purpose of this Exercise is to evaluate Gordon's case for dividend policy as a determinant of corporate value and capital cost in a wider context by introducing the comprehensive critique of Franco Modigliani and Merton H. Miller (MM henceforth) to the debate. Since 1958, their views on the *irrelevance of financial policy* (which includes dividend policy) based on their Nobel Prize winning economic "law of one price" and a wealth of empiricism has proved to be a watershed for the development of modern finance.

**The Theoretical Background**

According to MM, dividend policy is not a determinant of share price in reasonably efficient markets because dividends and retentions are *perfect economic substitutes*.

- If shareholders forego a dividend to benefit from a retention-financed capital gain, they can still create their own home made dividends to match their consumption preferences by the sale of shares and be no worse off.
- If companies choose to distribute a dividend they can still fulfil their investment requirements by a new issue of equity, rather than use retained earnings, so that the effect on shareholders' wealth is also neutral.

Theoretically and mathematically, MM have no problem with Gordon's model under conditions of *certainty*. They too, support Fisher's Separation Theorem that share price is a function of profitable corporate investment (business risk) and not dividend policy (financial risk).But where MM depart company from Gordon is under conditions of *uncertainty*.

MM maintain that Gordon's model fails to discriminate between financial policy (financial risk) and investment policy (business risk). For example, an increase in the dividend payout ratio, without any additional finance, reduces a firm's reinvestment capability and *vice versa*.

Using the earlier notation for the dividend growth model:

$$P_0 = D_1 / K_e - g$$

Change $D_1$, you change b and as a consequence g = br also changes. And if $K_e$ also changes as Gordon hypothesises, MM legitimately ask our earlier question:

> How are the differential effects of dividend policy and investment policy on price ($P_0$) measured when all the right hand variables of the Gordon equation are affected?

Perhaps you recall from our previous exercise that this represented a real problem for Gordon and others, who empirically encountered what statisticians formally term *multicolinearity*.

MM also assert (quite correctly) that because uncertainty is *non-quantifiable*, it is logically impossible for Gordon to capitalise a *multi-period* future stream of dividends, where $K_{e1} < K_{e2} < K_{e3}$ etc., according to the investors' financial perception of the unknown. A *one-period* model, where $K_e$ reflects the firm's *current* investment opportunities (business risk) is obviously more appropriate.

Finally, according to MM, if shareholders do not like the financial risk of their dividend stream they can always sell their holdings. So, why revise $K_e$?

**The MM Model**

Unlike Gordon, MM define an *ex-div* share price using a *one period model*. Moreover, their shareholder return ($K_e$) equals the company's cut-off (discount) rate applicable to the business risk of its current investment policy.

$$P_0 = D_1 + P_1 / 1 + K_e$$

> For a given investment policy, a change in dividend policy cannot alter current share price. According to MM, the future *ex div* price increases by the reduction in the dividend and vice versa.

To see why, let us return to the data set for Jovi plc in the previous exercise where the company first pursues a dividend policy of *maximum distribution* with:

$$E_1 = D_1 = 10 \text{ pence in perpetuity and } K_e = 2.5\%$$

MM would define an equivalent price to Gordon:

$$P_0 = D_1 + P_1 / 1 + k_e = £0.10 + £4.00 / 1.025 = \mathbf{£4.00}$$

But now, let us assume that the company pursues a policy of *maximum retention* to finance future investment of equivalent risk and see where this takes us.

According to MM, if the cut-off rate for investment still equals $K_e$ then the *ex div* price rises by the corresponding fall in the dividend, leaving $P_0$ unchanged.

$$P_0 = D_1 + P_1 / 1 + k_e = £0 + £4.10 / 1.025 = \mathbf{£4.00}$$

**The Shareholders' Reaction**

You will recall that Gordon argues if dividends *fall*, the capitalisation rate should *rise*, causing share price to *fall*. However, MM maintain that both return and price should remain the same.

> If shareholders *do not like the heat, they can get out of the kitchen* by creating *home-made* dividends through the sale of either part, or all of their holdings.

To prove the point, assume you own a number of Jovi's shares (let us say, n = 10,000) with the company's initial policy of full distribution. From the previous section, it follows that:

$$nP_0 = nD_1 + nP_1 / 1 + K_e = £1,000 + £40,000 / 1.025 = \mathbf{£40,000}$$

Now assume the firm withholds all dividends for reinvestment. What do you do if your income requirements (consumption preferences) equal the non-payment of your dividend (£1,000)?

According to MM, there is no problem. The *ex div* share price increases by the reduction in dividends, so, your holding is now valued as follows, with no overall change:

$$nP_0 = nD_1 + nP_1 / 1 + K_e = £0 + £41{,}000 / 1.025 = \mathbf{£40{,}000}$$

However, you still need to satisfy your income preference for £1,000 at time period one.

So, MM would suggest that you sell 250 shares for £41,000 / 10,000 at £4.10 apiece.

You now have £1,025, which means that you can take the income of £1,000 and reinvest the balance of £25 on the market at your desired rate of return ($K_e$=2.5%). And remember you still have 9,750 shares valued at £4.10. To summarise your new equilibrium position:

Shareholding 9,750: Market value £39,975: Homemade Dividends £1,000: Cash £25

So, have you lost out? According to MM, *of course not*, because future income and value are unchanged:

| | £ |
|---|---|
| $nP_1$ = 9,750 x £4.10 | 39,975 |
| Cash reinvested at 2.5% | 25 |
| Total Investment | 40,000 |
| Total annual return at 2.5% | 1,000 |

**The Corporate Perspective**

Let us now turn or attention to what is now regarded as the *proof* of the MM dividend *irrelevancy* hypothesis. This is usually lifted *verbatim* from the mathematics of their original article and relegated to an Appendix in the appropriate chapter of most financial texts, with little if any numerical explanation. So, where do we start?

The MM case for *dividend neutrality* suggests that shareholders can create their own *home-made* dividends, if needs be, by selling part or all of their holdings at an enhanced *ex-div* price. For its part too, the firm can resort to new issues of equity in order to finance any shortfall in its investment plans.

To illustrate the dynamics, consider Jovi plc that now has a policy of maximum retention (nil distribution) and a dedicated investment policy, whose shares are currently valued at £4.00 with an *ex-div* price of £4.10 at time period one:

$$P_0 = D_1 + P_1 / 1 + K_e = £0 + £4.10 / 1.025 = £4.00$$

Assuming Jovi has one million shares in issue (n) we can then derive its market capitalisation of equity:

$$nP_0 = nD_1 + nP_1 / 1 + K_e = £0 + £4.1m / 1.025 = £4m$$

The firm now decides *to distribute all earnings as dividends* (10 pence per share on one million issued). If investment projects are still to be implemented, the company must therefore raise new equity capital equal to the proportion of investment that is no longer funded by retained earnings. From the MM proof:

$$mP_1 = nD_1 = £100{,}000$$

Based on all the shares outstanding at time period one, $(n + m)\ P_1$, we can rewrite the equation for the total market value of the original shares in issue as follows:

$$nP_0 = [nD_1 + (n + m)P_1 - mP_1] / 1 + K_e$$

And because $mP_1 = nD_1$, this simplifies to the *fundamental* equation of their proof containing *no dividend term*.

$$nP_0 = (n + m)\ P_1 / 1 + K_e = (nP_1 + £100{,}000) / 1.025 = £4m$$

Since there is also only one unknown in the equation (i.e. $P_1$) then dividing throughout by the number of shares originally issued (n = one million).

$$P_0 = (P_1 + £0.10) / 1.025 = £4.00$$

And rearranging terms and solving for $P_1$:

$$P_1 = £4.00$$

Thus, as MM hypothesise:

- The *ex-div* share price at the end of the period ($P_1$) *falls* from its initial value of £4.10 to £4.00, which is exactly the same as the 10 pence *rise* in dividend per share ($D_1$) leaving $P_0$ *unchanged.*
- Because the dividend term has completely disappeared from their value equation, it is impossible to conclude that share price is a function of dividend policy.

### The MM Dividend Hypothesis: A Practical Illustration

To confirm the logic of the MM hypothesis yourself, let us modify Jovi's previous *nil* distribution policy to assess shareholder and corporate implications if management now adopt a policy of *partial* dividend distribution, say 50 per cent? So we begin with:

$$P_0 = (0 + £4.10) / 1.025 = £4.00$$

And from our data set, we know the company now intends to pay a dividend of 5 pence per share next year on one million currently issue. Without compromising its investment policy,

$$P_0 = (0.10 + £4.00) / 1.025 = £4.00$$

**Required:**

Explain why the firm's equity value is independent of its dividend payout ratio.

**An Indicative Outline Solution**

Our second exercise provides an opportunity to evaluate the role of investment and financial criteria that underpin the normative objective of shareholder wealth maximisation under conditions of certainty and uncertainty. Our reference point is the Gordon-MM controversy concerning the determinants of share price and capital cost in an all-equity firm. Are dividends and retentions *perfect substitutes,* leaving shareholder wealth and the corporate cut-off rate for investment unaffected by changes in dividend distribution policy?

**Points to Cover**

1. The Shareholders' Reaction
   The MM case for *dividend neutrality* suggests that if a firm *reduces* its dividend payout, then shareholders can create their own *home-made* dividends by selling part or all of their holdings at an enhanced *ex-div* price. But in our question, the company has *increases* its dividend payout ratio. So, do the shareholders have a problem?

2. Dividend Irrelevancy
   For a given investment policy, a change in dividend policy (either way) does not alter current share price. The future *ex div* price falls by the rise in the dividend for a given investment policy of equivalent business risk and *vice versa*, leaving the current *ex div* price unchanged.

   Using our data set, where Jovi pursues an initial policy of *nil distribution* and $K_e = 2.5\%$.

   $$P_0 = D_1 + P_1 / 1 + K_e = £0 + £4.10 / 1.025 = £4.00$$

   But now assume that the firm pursues a policy of 50 per cent retention to reinvest in projects of equivalent business risk (i.e. $K_e = 2.5$ per cent). MM would define:

   $$P_0 = D_1 + P_1 / 1 + K_e = £0.05 + £4.05 / 1.025 = £4.00$$

3. The Corporate Perspective
   For its part too, Jovi can resort to new issues of equity in order to finance any shortfall in investment plans. To illustrate, consider the company's original policy of *nil* distribution but a dedicated investment policy, with shares currently valued at £4.00 but at £4.10 next year

   $$P_0 = D_1 + P_1 / 1 + K_e = £0 + £4.10 / 1.025 = £4.00$$

Management now decide to distribute 50 per cent of corporate earnings as dividends (5 pence per share on one million shares currently in issue). If investment projects are still to be implemented, the company must therefore raise new equity equal to the proportion of investment that is no longer funded by retained earnings. From our MM proof:

$$mP_1 = nD_1 = £50{,}000$$

The substitution of this figure into the MM equation for the total market value of the original shares, based on all shares outstanding at time period one, equals:

$$nP_0 = [nD_1 + (n + m)P_1 - mP_1] / 1 + K_e$$

And because $mP_1 = nD_1$, the MM proof simplifies to an equation with *no dividend term*.

$nP_0 = (n + m)\ P_1 / 1 + K_e = (nP_1 + £50{,}000) / 1.025 = £4m$

Since there is only one unknown in this equation ($P_1$) then dividing through by the number of shares originally in issue (n = one million) and solving for $P_1$:

$$P_0 = (P_1 + £0.05) / 1.025 = £4.00$$
$$P_1 = £4.05$$

So, as MM hypothesise; the *ex-div* share price at the end of the period has fallen from its initial value of £4.10 to £4.05, which is exactly the same as the 5 pence rise in dividend per share, leaving $P_0$ unchanged.

## Summary and Conclusions

MM criticise the Gordon growth model under conditions of uncertainty from both a *proprietary* and *entity* perspective by focussing on home-made dividends and corporate investment policy, respectively.

According to MM, the current value of a firm's equity is independent of its dividend distribution policy, or alternatively its retention policy, because they are *perfect economic substitutes:*

The quality of earnings (business risk), rather than how they are packaged for distribution (financial risk), determines the shareholders' desired rate of return and management's cut-off rate for investment (project discount rate) in an all-equity firm and hence its share price.

Consequently, dividend policy is a *passive residual*, whereby unused funds are returned to shareholders because management has failed in their search for new investment opportunities, which at least elicit project ENPVs that maintain shareholder wealth intact.

# 6 Debt Valuation and the Cost of Capital

## Introduction

Chapter Six of our *SFM* text explains why corporate borrowing is attractive to management. Interest rates on debt are typically lower than equity yields. Debt (bond) holders accept lower returns than shareholders because their investment is less risky. Unlike dividends, interest is a *guaranteed* prior claim on profits. In the event of liquidation, bond holders like other *creditors* are also paid from the sale of any assets before shareholders. Finally, in many countries, interest payments on debt (unlike dividends) also qualify for corporate *tax relief*, reducing their *real* cost to the firm and widening the yield gap with equity still further.

> The introduction of borrowing into the corporate financial structure, termed capital *gearing* or *leverage*, can therefore lower the overall return (cut-off rate) that management need to earn on new investments. Consequently, the ENPV of geared projects should be greater than their all equity counterparts, producing a corresponding increase corporate wealth.

Our first exercise therefore reviews the fiscal benefits conferred on companies that issue corporate bonds (debentures) whilst the second deals with the derivation of an overall cost of capital from a combination of debt and equity as a managerial discount rate for project appraisal.

Because the development of the real world equations required to answer the first question are quite complex, you might need to refer back to their origins in Chapter Six of the *SFM* text. To aid cross-referencing, I have applied the original number to the equations where appropriate.

## Exercise 6.1: Tax-Deductibility of Debt and Issue Costs

If management can generate sufficient taxable profits to claim the tax relief on debt interest, the higher the rate of corporation tax, the greater the fiscal benefit conferred on the company through issuing debt, rather than equity, to finance investment. To prove the point, in the *SFM* text we defined the price of *irredeemable* debt incorporating the tax effect by using the PV model for the capitalisation of a perpetual annuity.

(6) $P_0 = I(1-t) / K_{dt}$

Rearranging terms, the "real" cost of debt for the company after tax:

(7) $K_{dt} = I(1-t) / P_0$

And because the investors' *gross* return ($K_d$) equals the company's cost of debt before tax, it follows that with a tax rate (t) we can also rewrite Equation (7) as follows;

(8) $K_{dt} = K_d (1-t)$

In a world of corporate taxation, the capital budgeting implications for management are clear.

(9) $K_{dt} < K_d$

To maximise corporate wealth, the post-tax cost of debt should be incorporated into any overall discount rate as a cut-off rate for investment.

Turning to *redeemable* debt, the company still receives tax relief on interest but often the redemption payment is not allowable for tax. To calculate the post-tax cost of capital it is necessary to determine an IRR that incorporates tax relief on interest alone. Thus, we derive $K_{dt}$ in the following *finite* equation:

(10) $P_0 = \sum_{t=1}^{n} I(1-t) / (1+K_{dt})^t + (P_n / 1 + K_{dt})^n$

> Irrespective of whether debt is irredeemable or redeemable, its tax adjusted cost of ($K_{dt}$) is the IRR that represents the true corporate cost of new debt issues. If the ENPV of a prospective debt-financed project discounted at this IRR is positive, then its return will exceed the cost of servicing that debt and management should accept it.

**Taxation Lags and Issue Costs**

The introduction of tax *bias* into our analysis of debt costs is only one real world adjustment. There are others, namely the *timing* of the tax benefit, set against the actual cost of *issuing* debt.

As we explained in *SFM*, corporate taxation might not be payable until well after profits are earned. We can therefore introduce greater realism into our calculations by incorporating a *time lag* associated with the interest set-off against corporate tax liability. Although this delay reduces the present value of the tax deduction to the company, the *net* cost of corporate debt will still be lower than the *gross* return to the investor.

Let us assume a *one-year* time lag between the payment of annual interest and the receipt of the tax benefit. The post-tax cost of redeemable debt, $K_{dt}$ can be found by solving for the IRR in the following value equation:

$$P_0 = \sum_{t=1}^{n} I / (1+K_{dt})^t \;+ (P_n / 1 + K_{dt})^n \;- \sum_{t=2}^{n+1} I.t / (1+K_{dt})^t$$

Thus, the value of debt is equal to the discounted *pre-tax* cash flows on the right-hand side of Equation (10), *less* the discounted sum of tax benefits from the *second* year of issue to the year *after* redemption (the final term on the right hand side of the above equation).

Of course, in the real world, the "real" price of loan stock and marginal cost of debt to the company is offset by issue costs, which can represent between three and six per cent of the capital raised.

This is best understood if we first substitute issue costs (C) into the cost of *irredeemable* debt in a *taxless* world (Equation 5 in *SFM*). The denominator of the equation is reduced by the issue costs, so the corporate cost or debt rises.

(13) $K_d = I / P_0 (1–C)$

If we now assume that interest is tax deductible (with no time lag) the post-tax cost of debt originally given by Equation (7) also rises.

(14) $K_{dt} = I (1\text{-}t) / P_0(1–C)$

But what if issue costs, as well as interest payments, are also tax deductible?

Using redeemable debentures, let us assume a one-year time lag for tax-deductibility associated with: *initial* issue costs and *annual* interest. Substituting this fiscal policy into the previous *time lag* equation should produce a lower corporate tax bill.

$$P_0 - \{C - [C\,(1-t)] / 1 + K_{dt}]\} = \sum_{t=1}^{n} I / (1+K_{dt})^t + (P_n / 1 + K_{dt})^n - \sum_{t=2}^{n+1} I.t / (1+K_{dt})^t$$

Indeed this debt valuation equation reveals how the tax deductibility associated with issue costs (the discounted left-hand term in brackets) works in the company's favour, just like tax relief on interest (the final right-hand term of the equation).

To comprehend the complexities of the previous post-tax, issue cost equation and confirm the difference between an investor's gross return and the company's after-tax cost of debt capital, consider the following information.

The Sambora Company intends to issue a new fifteen year corporate bond in £100 blocks at a coupon rate of 10 per cent with a redemption premium of 20 per cent. Issue costs are £3.00 per cent. The corporate tax rate is 50 per cent. Fiscal relief is staggered by one year.

**Required:**

1. Calculate the investors *yield to redemption*.
2. Calculate the company's *post-tax* cost of debt.

**An Indicative Outline Solution**

1. **The Redemption Yield**
   The investor yield to maturity solves for $K_d$ using Equation (5) from the *SFM* text. Annual interest payments and the redemption price are discounted back to a present value as follows:

$$P_0 = 100 = 12 / (1+r) + 12 / (1+r)^2 \ldots + \ldots 12 / (1+r)^{15} + 120 / (1+r)^{15}$$

   The IRR of the equation (yield to redemption) is approximately 12.5 per cent per annum.

2. **The Corporate Cost of Issue**
   With regard to the company, the cost of debt is lower than the cost to its clientele because issue costs and interest payments are tax deductible.

   The value of any £100 bond allowing for *net* transaction costs (the difference between £3.00 and the discounted 50 per cent tax relief on £3.00) is equal to the discounted interest payments from year one to fifteen less the tax deductible interest benefit of £6 per annum, discounted from year two through sixteen. Thus, using the time-lag equation that incorporates issue costs:

$$P_0 - \{C - [C\,(1-t) / 1+ K_{dt}]\} = \sum_{t=1}^{n} I / (1+K_{dt})^t + (P_n / 1 + K_{dt})^n - \sum_{t=2}^{n+1} I.t / (1+K_{dt})^t$$

$$100 - [3 - (1.5 / 1+ K_{dt})] = \sum_{t=1}^{n=15} 12 / (1+K_{dt})^t + 120 / 1 + K_{dt})^{15} - \sum_{t=2}^{n=16} 6 / (1+K_{dt})^t$$

   And solving for the IRR, we find that the corporate post-tax cost of debentures ($K_{dt}$) is approximately 7.4 percent per annum (compared with 12.5 per cent for investors).

## Exercise 6.2: Overall Cost (WACC) as a Cut-off Rate

With your knowledge of equity and debt valuation and their component costs we are now in a position to combine them to derive a company's weighted average cost of capital (WACC) as an overall cut-off (discount rate) for investment. Consider the following information:

The summarised Balance Sheet of Winehouse plc is as follows ($m).

| | | | |
|---|---|---|---|
| Ordinary Share Capital | 1,600 | Fixed Assets | 2,800 |
| Reserves | 800 | Net Current Assets | 200 |
| Debentures | 600 | | |
| Totals | 3,000 | | 3,000 |

Two proposals have been placed before the Board by the new Finance Director, each requiring an initial investment of $600,000 and the one piece of vacant land that the company has available, so that only one investment can be chosen.

Project I will generate net cash flows of $240,000 per annum for the first three years of its life and $100,000 per annum for the remaining two. Project II will generate net cash flows of $200,000 per annum during its life, which is also five years. Neither project has any residual value but the first project is regarded as the less risky of the two. There is £$80,000 of internally generated funds available and the remainder will have to be raised through the issue of ordinary shares and loan stock. However, Winehouse wishes to maintain its original capital structure. The current equity yield is 15% but a new issue of ordinary shares at $5 per share will result in net proceeds per share of $4.75. It is also envisaged that 8% bonds can be sold at par. The company has a marginal corporate tax rate of 25%.

**Required:**

1. Derive the marginal WACC applicable to each investment.
2. Determine the NPV of each project with an explanation of which (if either) maximises wealth.
3. Summarise the conditions that must be satisfied to validate WACC as a cut-off rate for investment.

**An Indicative Outline Solution**

The calculation of both projects' NPV requires the derivation of a discount rate, based upon the mathematical concept of a *weighted average* applied to the formulation of a company's *WACC* as an appropriate cut-off rate for investment. For example, with only two sources of capital (equity and debt say) and using standard notation, a general formula for WACC is given by:

$$K = K_e(V_E/V) + K_d(V_D/V)$$

Computationally, the component costs of capital are weighted as a proportion of the company's total market value and the results summated (i.e. added together).

1. **The WACC Computation**

   Incremental finance and capital costs using the desired capital structure.

| Finance | ($000) | Weight | Cost | Component | Derivation |
|---|---|---|---|---|---|
| Equity: Internal | 80 | 0.13 | 15.0% | 1.95% | given |
| External | 400 | 0.67 | 15.8% | 10.59% | less issue costs |
| Debt | 120 | 0.20 | 6.0% | 1.20% | post tax |
| Totals | 600 | 1.00 | WACC | 13.74% | |

2. **NPV Analysis**

   Rounding up the WACC to a 14 per cent discount rate, you should be able to derive the following NPVs for the two projects using the now familiar present value of the summation of discounted cash flows minus the cost of the investment.

$$[(PV@\ 14\%) - I_0] = NPV = \$70{,}200 \text{ for Project 1} < \$88{,}000 \text{ for Project 2}$$

Assuming that the normative objective of Winehouse is shareholder wealth maximisation, then a NPV maximisation approach to project appraisal means that Project 2 should be accepted if the investments are mutually exclusive and capital is rationed. Note, however, that the first project is less risky. But is this important, given the small scale of the projects relative to the company's overall size? The *marginal* nature of the projects also leads one to ask why the firm wishes to retain its existing capital structure when debt is the cheapest source of finance, only costing the company 6 per cent after tax?

3. **The WACC Assumptions**
To maximise NPV, it is a function of management to establish a discount rate, having acquired capital in the most efficient (least costly) manner. If funds are acquired from a miscellany of efficient sources to finance projects, it also seems reasonable to assume that the derivation of a marginal WACC should represent the optimum discount rate.

> In an efficient capital market, optimum projects should produce returns in excess of their minimum WACC at a maximum NPV that not only exceeds shareholders' expectations of a dividend and capital gain but also the returns required by all other providers of capital (Fisher's Theorem again).

However, the use of WACC as an appropriate discount rate in project appraisal must satisfy the following conditions:

- The selected investment is *homogenous* with respect to the overall business risk that already confronts the firm; otherwise the returns required by investors will change.
- The capital structure is reasonably *stable*; otherwise the weightings applied to the component costs of the WACC calculation will be invalid.
- As a consequence, the investment should be *marginal* to the firm's existing operations.

With regard to the calculation itself, the overall cost of capital is found *via* the identification of all types of capital used (including opportunity costs) which are weighted according to the existing capital structure) and then summated to produce the WACC.

- The weights are based on the market value of securities, rather than their book values, so as to reflect current rather than historical costs.
- The costs of equity are the returns expected by the shareholders on their funds invested in the business (reserves and new issues) adjusted for issue costs.
- The cost of debt is the current market rate of interest net of tax relief, which can be derived from existing borrowing.

## Summary and Conclusions

In Chapter One our study of strategic financial management began with a hypothetical explanation of a company's overall cost of capital as an investment criterion designed to maximise shareholder wealth. By Chapter Five we demonstrated that an *all equity* company should accept capital projects using the marginal cost of equity as a discount rate, because the market value of ordinary shares will increase by the project's NPV.

In this chapter we considered the implications for a project discount rate if funds are obtained from a variety of sources other than the equity market, each of which requires a rate of return that may be unique.

For the purpose of exposition, we analysed the most significant alternative to ordinary shares as an external source of funding, namely redeemable and irredeemable loan stock. We observed that corporate borrowing is attractive to management because interest rates on debt are typically lower than equity yields. The impact of corporate tax relief on debenture interest widens the gap further, although the tax-deductibility of debt is partially offset by the costs of issuing new capital, which are common to all financial securities.

In this newly *leveraged* situation, the company's overall cost of capital (rather than its cost of equity) measured by a *weighted average cost of capital* (WACC) seems a more appropriate investment criterion. However, we observed that a number of conditions must be satisfied to legitimise its use as a project discount rate. In the next chapter we shall examine these further.

# 7 Debt Valuation and the Cost of Capital

## Introduction

For the purpose of exposition, the derivation of a company's weighted average cost of capital (WACC) in Chapter Six was kept simple. Given financial management's strategic objective is to maximise the market value of ordinary shares, our analysis assumed that:

- The value attributed by the market to any class of financial security (debt or equity) is the PV of its cash returns, discounted at an opportunity rate that reflects the financial risk associated with those returns.
- The NPV of a project, discounted at a company's WACC (based on debt plus equity) is the amount by which the market value of the company will increase if the project is accepted; subject to the constraint that acceptance does not change WACC.

We specified *three* necessary conditions that underpin this constraint and justify the use of WACC as a cut-off rate for investment.

- The project has the same business risk as the company's existing investment portfolio.
- The company intends to retain its existing capital structure (i.e. financial risk is constant).
- The project is small, relative to the scale of its existing operations.

Yet, we know that even if business risk is *homogenous* and projects are *marginal*, the financial risk of future investments is rarely *stable*. As the global meltdown of 2007 through to 2009 confirms, the availability of funds (debt and equity) is *the limiting factor*. The component costs of project finance (and hence WACC) are also susceptible to change as *external* forces unfold.

> So, let us develop a *dynamic* critique of the overall cost of capital (WACC) and ask ourselves whether management can increase the value of the firm, not simply by selecting an optimal *investment*, but also by manipulating its *finance*. If so, there may be an optimal capital structure arising from a debt-equity *trade-off*, which elicits a least-cost combination of financial resources that minimises the firm's WACC and maximises its total value.

In the summary to Chapter Seven of the *SFM* text we touched on the case for and against an optimal capital structure and WACC based on "traditional" theory and the MM economic "law of one price" respectively. The second exercise will pick up on these conflicting analyses in detail. Specifically, we shall examine the MM *arbitrage* proof, whereby investors can profitably trade securities with different prices between companies with different leverage until their WACC and overall value are in *equilibrium*.

Unlike the traditionalists, MM maintain that the equilibrium value of any company is *independent* of its capital structure and derived by capitalising expected project returns at a *constant* WACC appropriate to their class of business risk. Yet both theories begin with a common assumption. Because of higher financial risk the cost of equity is higher than the cost of debt and rises with increased leverage (gearing).

So, before we analyse why the two theories part company, our first exercise will explain how increased gearing affects shareholder returns by graphing the relationship between earnings yields and EBIT (net operating income) when firms incorporate cheaper debt into their capital structure.

Like our approach to the questions in Chapter Five, we shall accompany each of the current exercises with an exposition of the theories required for their solution, where appropriate. And because of their complexity, we shall (again) develop a data set throughout Exercise Two using a "case study" format. To tackle its sequential exposition (just like Chapter Five) you may need to refer back and forth, supplementing your readings with any other texts, purchased or downloaded from the internet.

## Exercise 7.1: Capital Structure, Shareholder Return and Leverage

To assess the impact of a changing capital structure on capital costs and corporate values, let us begin with a fundamental assumption of capital market theory, which you first encountered in Chapter One, namely that investors are *rational* and *risk averse*. Companies must offer them a return, which is inversely related to the probability of its occurrence. Thus, the crucial question for financial management is whether a combination of stakeholder funds, related to the earnings capability of the firm, can minimise the risk which confronts each class of investor. If so, a firm should be able to minimise its own discount rate (WACC) and hence, maximise total corporate value for the mutual benefit of all.

We know from previous Chapters that *total risk* comprises two inter-related components with which you are familiar, *business risk* and *financial risk*. So, even when a firm is financed by equity alone, the pattern of shareholder returns not only depends upon periodic post-tax profits (business risk) but also managerial decisions to withhold dividends and retain earnings for reinvestment (financial risk). As we explained in Chapter Five, if rational (risk averse) investors prefer dividends now, rather than later, the question arises as to whether their equity capitalisation rate is a positive function of a firm's retention ratio. In otherwords, despite the prospect of a capital gain, does a "bird in the hand" philosophy elicit a premium for the financial risk associated with any diminution in the dividend stream? If so, despite *investment* policy, corporate *financial* policy must affect the overall discount rate which management applies to NPV project analyses and therefore the market value of ordinary shares.

When a firm introduces debt into its capital structure we can apply the same logic to arrive at similar conclusions. Financial policies *matter* because the degree of leverage (like the dividend payout ratio) determines the level of financial risk that confronts the investor.

**The Theoretical Background**

Initially, when a firm borrows, shareholder wealth (dividend plus capital gain) can be increased if the effective cost of debt is lower than the original earnings yield. In efficient capital markets such an assumption is not unrealistic:

- Debt holders receive a guaranteed return and in the unlikely event of liquidation are usually given security in the form of a prior charge over the assets.
- From an entity viewpoint, debt interest qualifies for tax relief.

You should note that the productivity of the firm's resources is unchanged. Irrespective of the financing source, the same overall income is characterised by the same degree of business risk. What has changed is the mode of financing which increases the investors' return in the form of EPS at minimum financial risk. So, if this creates demand for equity and its market price rises proportionately, the equity capitalisation rate should remain *constant*. For the company, the beneficial effects of cheaper financing therefore outweigh the costs and as a consequence, its overall cost of capital (WACC) falls and total market value rises.

Of course, the net benefits of gearing cannot be maintained indefinitely. As a firm introduces more debt into its capital structure, shareholders soon become exposed to greater financial risk (irrespective of dividend policy and EPS), even if there is no realistic chance of liquidation. So much so, that the demand for equity tails off and its price begins to fall, taking total corporate value with it. At this point, WACC begins to rise.

The increased financial risk of higher gearing arises because the returns to debt and equity holders are interdependent stemming from the same investment. Because of the contractual obligation to pay interest, any variability in operating income (EBIT) caused by business risk is therefore transferred to the shareholders who must bear the inconsistency of returns. This is amplified as the gearing ratio rises. To compensate for a higher level of financial risk, shareholders require a higher yield on their investment, thereby producing a lower capitalised value of earnings available for distribution (i.e. lower share price). At extremely high levels of gearing the situation may be further aggravated by debt holders. They too, may require ever-higher rates of interest per cent as their investment takes on the characteristics of equity and no longer represents a prior claim on either the firm's income or assets.

Even without increasing the interest rate on debt, the impact of leverage on shareholder yields can be illustrated quite simply. Consider the following data:

| **Company** | | **Ulrich (£ million)** | | | **Hammett (£ million)** | | |
|---|---|---|---|---|---|---|---|
| MARKET VALUES | | | | | | | |
| | Equity | | 100 | | | 60 | |
| | Debt | | – | | | 40 | |
| | Total | | 100 | | | 100 | |
| NET OPERATING INCOME | | | | | | | |
| | EBIT | 8.0 | 10.0 | 12.0 | 8.0 | 10.0 | 12.0 |
| | Interest (10%) | – | – | – | 4.0 | 4.0 | 4.0 |
| | EBT | 8.0 | 10.0 | 12.0 | 4.0 | 6.0 | 8.0 |
| | Corporation Tax (25%) | 2.0 | 2.5 | 3.0 | 1.0 | 1.5 | 2.0 |
| | EAT | 6.0 | 7.5 | 9.0 | 3.0 | 4.5 | 6.0 |
| | Earnings Yield (%) | 6% | 7.5% | 9% | 5% | 7,5% | 10% |

The two companies (Ulrich and Hammett) are identical in every respect except for their methods of financing. Ulrich is an all-equity firm. Hammett has £40 million of 10 per cent debt in its capital structure. A comparison of net operating income (EBIT) and shareholder return (earnings yield) is also shown if business conditions deviate 20 per cent either side of the norm.

What the table reveals is that the returns to ordinary shareholders in the all-equity company only fluctuate between 6 per cent and 9 per cent as EBIT (business risk) fluctuates between £8 million and £12 million. However, for the geared company the existence of a fixed interest component amplifies business risk in terms of the total risk borne by the ordinary shareholder. Despite the benefits conferred on Hammett and its shareholders by the tax deductibility of debt, the greater range of equity returns (5–10 per cent) implies greater financial risk.

Thus, if shareholders act rationally and business prospects are poor, they may well sell their holdings in the geared company, thereby depressing its share price and buy into the all-equity firm causing its price to rise.

Our preceding discussion suggests that for a given level of earnings a company might be able to trade the costs and benefits of debt by a combination of fund sources that achieve a lower WACC and hence a higher value for equity. To implement this strategy, however, management obviously need to be aware of shareholder attitudes to its existing financial policy and those of competitors under prevailing economic conditions. Even "blue chip "companies with little chance of liquidation are not immune to financial risk,

**Required:**

Use the previous data for Ulrich and Hammett to:

1. Graph the relationship between the respective earnings yield (vertical axis) and EBIT (horizontal axis) and establish the *indifference point* between their shareholder clienteles.
2. Summarise what your graph illustrates concerning shareholder preferences.

**An Indicative Outline Solution**

From the raw data you should have observed that if shareholders require a 7.5 per cent return and the EBIT (NOI) of both companies equals 10 million, they would be indifferent to investing in either, irrespective of current financial policies. By plotting a graph, however, you can also see that the relationship between earnings yield and EBIT is positive and linear for both companies but *different*. For the all-equity firm it is less severe, with a shareholder's return of zero corresponding to an EBIT figure of zero that passes through the origin in Figure 9.1. For the geared company, the EBIT figure which equates to a zero earnings yield intersects the horizontal axis at the value of 10 percent debenture interest payable (£4 million) and rises more steeply.

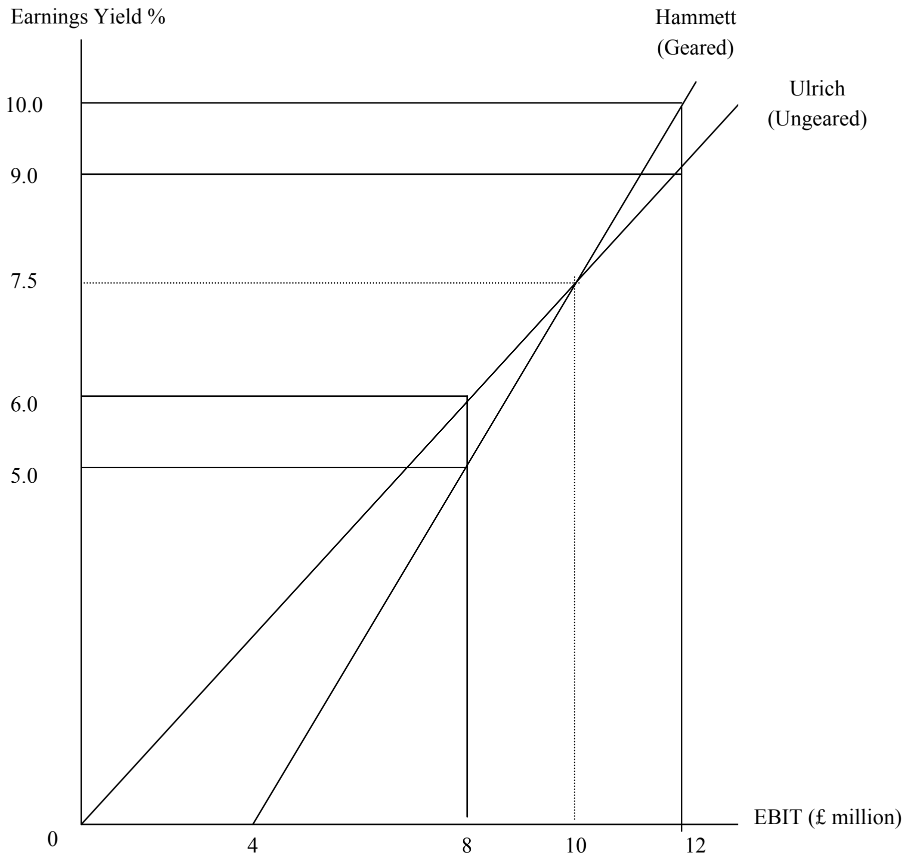


**Figure 7.1:** Capital Gearing and the Relationship between EBIT and Earnings Yield

The intersection of the two straight lines represents the point of indifference between the two companies. To the left of this point, shareholders would prefer to invest in Ulrich (ungeared) since they receive a better return for a lower level of EBIT. To the right, they would prefer Hammett (geared) for the same reasons. What we are observing is that leverage, which here means the incorporation of 10 per cent loan stock into a firm's capital structure, increases shareholders' sensitivity to changes in EBIT (business risk) and therefore the financial risk associated with equity; hence the steepness of the line.

## Exercise 7.2: Capital Structure and the Law of One Price

The previous exercise illustrates why rational risk averse investors prefer the ordinary shares of higher geared companies when economic conditions are good or improving but switch to lower geared firms when recession looms. Both strategies represent a rational risk-return trade off because:

- Ordinary shares represent a more speculative investment when there is a contractual obligation on the part of the company to pay periodic interest on debt
- As a general rule, the higher the gearing and more uncertain a firm's overall profitability (EBIT) the greater the fluctuation in dividends plus reserves.

As we mentioned earlier, the returns to debt and equity holders are interdependent, stemming from the same resources. So, what we are observing is the transfer of business risk to shareholders who must bear the inconsistency of returns as the firm gears up. Thus, it would seem that management should finance its investments so that the shareholders, to whom they are ultimately responsible, receive the highest return for a given level of earnings and risk. And this is where MM disagree with traditional theorists.

**The Traditional Theory of Capital Gearing and WACC**

Traditionalists believe that if a firm substitute's lower-cost debt for equity into its capital structure WACC will fall and value rise to a point of indebtedness where both classes of investor will require higher returns to compensate for increasing financial risk. Thereafter, WACC rises and value falls, suggesting an optimum level of gearing that minimises WACC and maximises value.

Figure 7.2 sketches these phenomena using the notation from our *SFM* text. The debt-equity ratio ($V_D$/ $V_E$) is plotted along the horizontal axes of both diagrams. The costs of both types of capital (setting $K_d < K_e$) are given on the vertical axis of the upper graph. The vertical axis on the lower graph plots total market value ($V=V_E+V_D$).To keep the analysis simple $K_d$ is held constant and its tax deductibility is ignored. Our aim is not to develop a real world model (more of which later) but to illustrate the basic relationships between capital costs, corporate value and leverage.

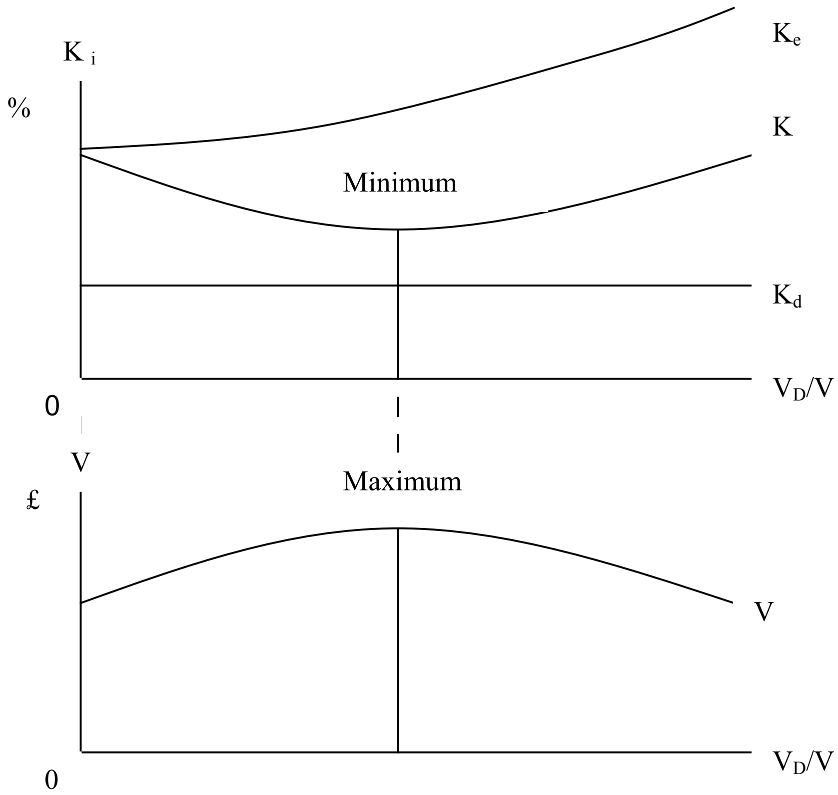


**Figure 9.2:** Traditional Theory with a Constant Cost of Debt in a Taxless World

Figure 9.2 confirms the traditional view that WACC is characterised by a U-shaped average cost curve K (familiar to economists).This is because the benefits of cheaper debt finance ($K_d<K_e$) are eventually offset by an increasing cost of equity as the firm gears up.

Turning to total market value V, (equity plus debt) if we define the relationship:

(1) $V = NOI / K$

where:

$V = V_E + V_D$ = total market value

$V_E$ = market value of equity

$V_D$ = market value of debt

NOI = net operating income (earnings before interest)

$K = WACC,$

$= K_e (V_E / V_E + V_D) + K_d (V_D / V_E + V_D)$

$= K_e (V_E / V) + K_d (V_D / V)$

$K_e$ = cost of equity

$K_d$ = cost of debt

We now observe an *inverse* relationship exists between V and K, given NOI. As one rises, the other falls and *vice versa*. Thus, the lower graph of Figure 9.2 illustrates that relative to the degree of leverage, the total market value of the firm has an *inverted* U-shaped function. As K (WACC) responds to changes in the gearing ratio and the rising cost of equity, V presents us with a mirror image. So, according to traditional theory, if companies borrow at an interest rate lower than their returns to equity, the implications for financial management are clear.

> For a given investment policy, there exists an optimal financial policy (debt-equity ratio) which defines a least-cost combination of financial resources.
>
> At the point where overall cost of capital is minimised, total corporate value is maximised and so is the market value of ordinary shares.

**The MM Cost of Capital Hypothesis**

Like much else in finance, the traditional case for an optimal capital structure did not arise from hard empirical evidence, or mathematical precision, but merely plausible assumptions concerning the cost of equity at different levels of gearing. But what if the overall relationship between the two is mistaken? Would an optimal WACC and corporate value still emerge?

To answer both these questions MM developed an alternative hypothesis, which produced two startling conclusions that confounded both traditional theorists and financial analysts.

> The total value of a firm represented by the NPV of an income stream discounted at a rate appropriate to its business risk, should be unaffected by shifts in financial structure.
>
> Any rational debt-equity ratio should also produce the same cut-off rate for investment (WACC).

Unlike many of their contemporaries, MM based their conclusions not on anecdote but *partial equilibrium* analysis, preceded by a number of rigorous assumptions, which they then substantiated by empirical research. The assumptions should be familiar, since they are based on *perfect markets* first outlined in Chapter One of the *SFM* text.

- Investors are rational.
- Information is freely available.
- Transaction costs are zero.
- Debt is risk-free.
- Investors are indifferent between corporate and personal borrowing.

MM also based their analysis on the traditional equation for total market value:

(1) V = NOI / K

However, where they disagree with traditional theory relates to their definition of WACC, which hinges on the behaviour of the equity capitalisation rate

MM reason that WACC (K) reflects the business risk associated with total earnings (NOI) rather than their financial risk, i.e. how they are packaged for distribution in the form of dividends, retentions or interest. Assuming that NOI is constant, they maintain that irrespective of the debt-equity ratio ($V_D$ / $V_E$) the company's WACC (K) and hence overall value (V) must be constant.

Based on their "economic law of one price" MM further reasoned that irrespective of leverage, because shares in similar companies cannot sell at different prices, two companies with the same total income and business risk will have the same total market value and WACC, even if their gearing ratios differ.

Expressed algebraically, if:

$V_1 = V_2$ = the value for two companies.
$I_1 = I_2$ = their common NOI.

The WACC for any company in the same risk class:

(2) $K = I_1/V_1 = I_2/V_2$

And because $K = K_e$ in the unlevered firm, the WACC for the geared company must also equal the cost of equity capital $K_e$ of the all equity firm.

Thus, if the cost of debt $K_d$ is constant (an assumption that MM later relax) all that needs to be resolved is the precise relationship between the rising cost of equity $K_e$ and the debt-equity ratio $V_D/V_E$ when a firm gears up. Is it *exponential*, as the traditionalists suggest (Figure 9.2) or not.

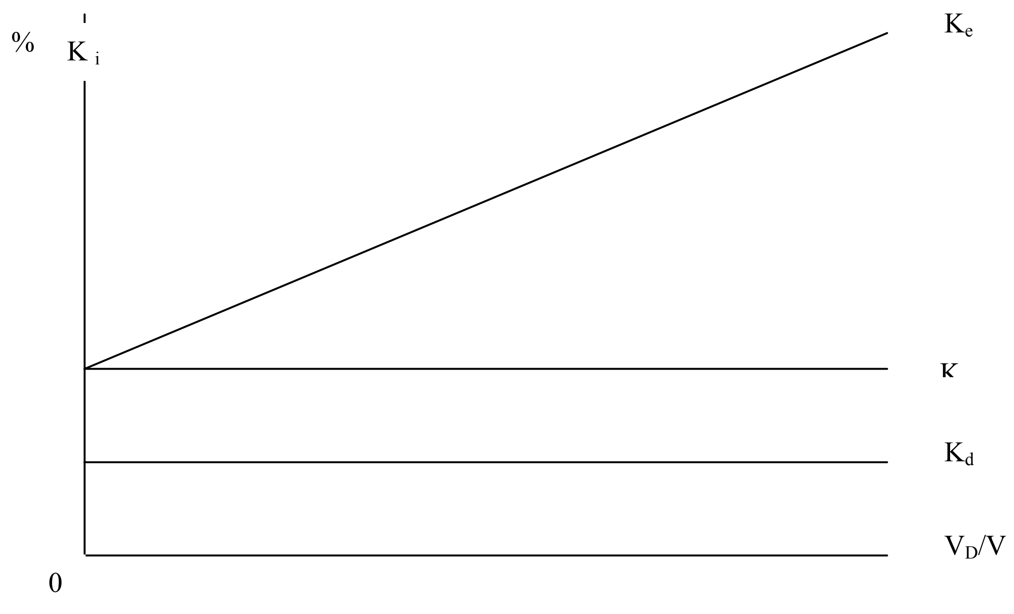


**Figure 9.3:** The MM Theory with a Constant Cost of Debt in a Taxless World

According to MM, if we ignore corporate tax and tax relief on interest, the equity capitalisation rate $K_e$ will still increase but not exponentially as the traditionalists believe. The rise *exactly offsets* the benefits of increasing the proportion of cheaper loan stock in a firm's capital structure leaving WACC unchanged. This *linear* relationship is sketched in the upper graph of Figure 9.3, which translates into the following equation.

(2) $K_{eg} = K_{eu} + [(V_D / V_E)(K_{eu} - K_d)]$

where:
$K_{eg}$ = the cost of equity in a geared company
$K_{eu}$ = the cost of equity in an ungeared company
$K_d$ = the cost of debt capital
$V_D$ = the market value of debt in the geared company
$V_E$ = the market value of equity in the geared company

$K_{eg}$ (leveraged) is equivalent to $K_{eu}$, the capitalisation rate for an all-equity stream of the same class of business risk, plus a premium related to financial risk. This is measured by the debt-equity ratio ($V_D/V_E$) multiplied by the spread between $K_{eu}$ and $K_d$.

The financial risk premium (the second term on the right of our preceding equation) causes equity yields to rise at a *constant* rate as compensation for financial risk when the firm gears up.

Since the WACC in companies of equivalent business risk is the same, irrespective of leverage, their total market value (V) will also be the same if the companies are identical in every respect except their gearing ratio. Thus:

(3) $V_U = V_G = V_E + V_D$

where:
$V_U$ = the market value of an ungeared all equity company
$V_G$ = market value of an identical geared company (equity plus debt)

The lower graph of Figure 9.3 plots constant value (V) against an increasing debt-equity ratio ($V_D/V_E$).

If WACC and overall corporate value are unaffected by leverage as MM hypothesise, the implication for strategic financial management are profound. As we mentioned in Chapter One of the *SFM* text, financial decisions (which include the dividend policy of Chapter Five, as well as gearing) are irrelevant to investment decisions (project valuation and selection).

**Reading and Review**

In a subject still dominated by the work of Modigliani and Miller it is important that you refer to their original articles if only to confirm what you read elsewhere.

MM's 1958 paper "The Cost of Capital, Corporation Finance and the Theory of Investment" sets out their original case for the irrelevance of financial structure to corporate valuation and capital cost (WACC) in a perfect capital market. Find it and skim through to get the broad thrust of their arguments (even if you find the mathematics complex). Then produce brief answers to the following questions before we move on to our second exercise.

a) There are three *propositions* advanced by MM. What are they and how are they proved?
b) How do MM's conclusions differ from a traditional view of capital structure in a taxless world where the cost of debt is constant?
c) Within the context of investment appraisal, what are the implications of MM's hypothesis for financial management?

**MM: A Review**

a) **The Propositions**

Using our own notation, the three propositions advanced by MM are:

Proposition I: Overall market value (V) is *independent* of the debt-equity ratio ($V_D/V_E$)

Proposition II: To offset financial risk, the equity capitalisation rate ($K_{eg}$) increases at a *constant* rate as $V_D/V_E$ rises, with the corollaries:

- K is unaffected by $V_D/V_E$
- $K = K_{eu}$ for an unlevered firm.

Proposition III: Shareholder wealth is maximised by *substituting* an equity capitalisation rate ($K_{eu}$) of an unlevered firm for the cut-off rate (K) of a levered firm.

MM then explain how:

(i) Proposition I can be proved by *arbitrage* (more of which later).
(ii) Proposition I can be used to prove Proposition II which states that K is unaffected by $V_D/V_E$.
(iii) Proposition III follows logically from Positions II and III, since market value equals equity value ($V = V_E$) and therefore $K = K_e$ in an unlevered firm.

b) **The Conclusions**

Even in a world of zero taxation with a constant cost of debt, a comparison of Figure 9.2 with Figure 9.3 reveals that MM's conclusions contrast sharply to a traditional view. WACC does not vary with gearing. There is no optimal debt-equity ratio and the market value of the firm remains constant. According to MM, the cost of equity capital is no longer an exponential function of increasing leverage. Given MM's contention that K is constant, $K_e$ rises *linearly* as $V_D/V_E$ increases.

c) **The Investment Implications**

| |
|---|
| If MM's hypothesis is correct, the "traditional" financial decisions that confront management when investment decisions include debt are eliminated. The net result is that WACC (the cut-off rate for investment) and total corporate value remain the same. Gearing is therefore irrelevant to project evaluation and shareholder wealth maximisation. |

**Proposition I and the Arbitrage Process**

Your reading should confirm that that the logic of MM's cost of capital hypothesis stems from their first proposition that corporate value is independent of capital structure because of *arbitrage*.

> Arbitrage occurs when investors sell financial securities to buy cheaper perfect substitutes, thereby depressing the price of the former and increasing the price of the latter, until their market prices are in equilibrium.

MM maintain that if a traditional view of capital structure were to exist it should only be a short-run *dis-equilibrium* phenomenon in perfect markets. Rational (risk-averse) *arbitrageurs* will respond quickly to prevent the existence of the two firms with identical risk and the same NOI from selling at different prices.

Shareholders in an over-valued company (what the traditionalists would call highly geared) will change its total value by selling shares in that company and buying shares in an under-valued (i.e. ungeared) company. In the process shareholders will even undertake personal borrowing to maximise their stake in the ungeared company at a level where their personal investment portfolios have the same degree of leverage as the overvalued firm.

As a result of what MM term *home-made leverage* (personal borrowing), investor income is increased at no greater financial risk. Eventually, through supply and demand forces, the price of shares in the overvalued company will fall, while that of the undervalued company will rise until no further financial advantage is gained. At this *equilibrium* overall market value, the overall cost of capital (WACC) for the two companies will also be the same.

For the mathematically minded, we could illustrate the whole arbitrage process by reference to formidable *algebraic* relationships that compare MM to a traditional view, in a tax-free world where the cost of debt is constant. For ease of exposition, however, we shall restrict our second exercise to a *numerical* example with a modest series of equations.

Let us begin with a series of *traditional* financial relationships between two firms (Elbow and Dimebag) that are identical in every respect, except for their capital structure (€000s).

| | Elbow (ungeared) | | Dimebag (geared) |
|---|---|---|---|
| **Distributable Earnings (No Tax)** | | | |
| NOI | 100 | = | 100 |
| Debt Interest (Kd = 5%) | - | | 10 |
| Shareholder Income | 100 | > | 90 |
| **Market Values** | | | |
| Equity (VE) | 1000 | > | 900 |
| Debt (VD) | - | | 200 |
| Total Value (V) | 1000 | < | 1,100 |
| **Capital Costs** | | | |
| Equity Yield (Ke) | 10% | = | 10% |
| Cost of Debt (Kd) | - | | 5% |
| WACC (K = NOI / V) | 10% | > | 9.09% |

**Required:**

1. Use the previous data to illustrate the benefits of arbitrage for an investor who currently owns 10 per cent of Dimebag's shares.
2. Summarise the effects of arbitrage as more investors enter the process.

**An Indicative Outline Solution**

From the data you should have observed what MM term *disequilibrium*. The total market value and WACC of equivalent companies differ. So, arbitrage is a profitable strategy for all investors in the geared firm.

1. **The Arbitrage Process**
   Now let us consider a series of arbitrage transactions for a single investor who holds 10 per cent of the equity in Dimebag (the higher valued geared firm) whose annual income is therefore €9,000 (€90,000 x 0.10).

1. She sells her total shareholding for €90,000 (10 per cent of €900,000) which reduces the financial risk of investing in the geared company to zero.
2. She now buys shares in Elbow (the ungeared, all-equity firm) but how much should she spend?

3. In order to compare like with like, it is important to hold the investor's exposure to financial risk at the same level as her original investment in Dimebag. With a €90,000 equity stake in that company, management presumably used this as collateral to borrow €20,000 of corporate debt on her behalf (i.e.10 per cent of €200,000). So, in a perfect capital market where private investors can borrow on the same terms as the company, she can substitute *homemade* leverage for *corporate* leverage to finance her new investment in the all-equity firm.
4. She borrows €20,000 at 5 per cent per annum, an amount equal to 10 per cent of the firm's debt.
5. As a result, the investor now has a total of €110,000 (€90,000 cash, plus €20,000 of personal borrowing) with which to purchase the ungeared shares in Elbow.
6. Because Elbow's yield is 10 per cent, the investor will receive an annual return of €11,000 (€110,000 × 0.10). However, she must pay annual interest on her personal loan (€20,000 × 0.05 = €1000).Therefore, her annual net income will be €10,000 (€11,000 – €1,000).

So, to conclude, is our investor better off? We can measure her change in income as follows:

| | € |
|---|---|
| Shareholder income in Elbow (ungeared) | 11,000 |
| Shareholder income in Dimebag (geared) | 9,000 |
| Change in income | 2,000 |
| Interest on borrowing (5%) | 1,000 |
| Net Gain from *Arbitrage* | 1,000 |

**The Arbitrage Process**

Thus, shareholder income has increased with no change in financial risk. The reason the investor has benefited is because the leveraged shares of Dimebag are overvalued relative to those of Elbow. If proof be needed, you should be able to confirm that the equity capitalisation rates for both firms originally equalled 10 per cent, despite differences in their total shareholder income.

2. **Summary**
As more investors enter the arbitrage process (trading shares to profit from disequilibrium) the equity value of geared firms will fall, whilst those of their ungeared counterparts will rise. To similar but opposite effect, their equity capitalisation rates will rise and fall respectively, until their overall cost of capital (WACC) is equal. Thus, MM's message to "traditionalists" is clear.

> Inequilibrium, shareholders will be indifferent to the degree of leverage and the arbitrage process becomes irrelevant to management's strategic evaluation of project investment and its wealth maximisation implications.

## Summary and Conclusions

We have considered whether a company can implement financial policies concerning capital structures that minimise weighted average cost of capital (WACC) and maximise total corporate value. Given your knowledge of equity valuation (Chapter Five) and the derivation of debt cost (Chapter Six) we focussed upon the controversial question of whether optimal financial decisions contribute to optimum investment decisions.

The traditional view states that if a firm trades lower-cost debt for equity, WACC will fall and value rise to a point of indebtedness where both classes of investor will require higher returns to compensate for increasing financial risk. Thereafter, WACC rises and value falls, suggesting an optimum capital structure.

In 1958, Modigliani and Miller (MM) discredited this view under the assumptions of perfect markets with no barriers to trade, by proving that WACC and total value are independent of financial policy. Based on the economic *law of one price*, they used *arbitrage* to demonstrate that close financial substitutes, such as two firms in the same class of business risk with identical net operating income (NOI), cannot sell at different prices; thereby negating financial risk.

The MM proof confounded the traditional investment community who argued that their assumption of perfect markets, particularly a neutral tax system without tax relief on debenture interest invalidated their conclusions. However, MM were the first to concede that an allowance for tax relief will reduce the cost of loan stock, lower WACC and increase total value as a firm gears up. The whole point of their hypothesis was to provide a *benchmark* to assess the impact of incorporating more realistic assumptions as a basis for more complex analyses. For example:

- Do personal as well as corporate fiscal policies affect capital structure?
- Are corporate borrowing and investment rates equal?
- How do investor returns behave with extreme leverage?
- Are management better informed than stock market participants?
- Do managerial objectives conflict those of investors
- And if so, do management prefer different sources of finance (think share options).

Unfortunately, we still have few definitive answers. The capital structure debate has ebbed and flowed since MM published their original hypothesis in 1958 with a surprising lack of consensus among academics, researchers and practitioners. To complicate matters further, historical research has obviously focussed on observable, modest (rational) debt equity ratios, rather than the extreme (irrational) leverage that has created global financial distress and bankruptcy since 2007.

To learn the lessons of the recent past, perhaps the debate will take a new turn. If so, real world management could learn from their mistakes by returning to first principles and revive MM's basic propositions on the irrelevance of financial policy. They provide a sturdy framework for rational investment. Moreover, their cost of capital hypothesis is entirely consistent with their 1961 dividend irrelevancy hypothesis covered in Chapter Five (for which there is considerable empirical support).

Thus, it seems reasonable to conclude that if we are to emerge from the current global, economic crisis "all singing from the same old song sheet".

- Corporate value should depend on the agency principle (Chapter One) characterised by an investor-managerial consensus on the level of earnings and their degree of risk, rather than the proportion distributed.
- Dividend and retention decisions should be irrelevant to the marketprice of a share (Chapter Five).
- The division of returns between debt and equity asa determinant of WACC and total corporate value should also be perfect substitutes.

**Reference**

1. Modigliani, F. and Miller, M.H., "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review,* Vol. XLVIII, No. 3, June, 1958.

# Part Four
The Wealth Decision

# 8 Shareholder Wealth and Value Added

## Introduction

Our study of Strategic Financial Management has revealed a series of controversial, theoretical relationships between shareholder wealth, dividend policy and the derivation of WACC as a cut-off rate for investment. Unfortunately, even if the differences between competing theories were resolved, there might still be no guarantee that real-world managerial self-interest would coincide with shareholder wealth maximisation. Time and again throughout the *SFM* text, when projects are being evaluated and modelled, we have used recent financial crises to prove the point.

In Chapter Eight we therefore explained how two American consultants, Joel Stern and Bennett Stewart have long sought to minimise any *principle-agency* problems for their corporate clients through the application of value added techniques.

According to Stern-Stewart, what companies require is an *internal*, incentive-based earnings driver, which shareholders can confirm from periodic *external* financial data to vet managerial performance. *Economic value added* (EVA) provides the internal metric. Moreover, they maintain that it is highly correlated to increases in shareholder wealth measured by the company's *market value added* (MVA).

So, how do they work?

## Exercise 8.1: Shareholder Wealth, NPV Maximisation and Value Added

Consider the Grohl Company that is currently committed to NPV maximisation in order to satisfy its overall shareholder wealth maximisation objective. The new Finance Director proposes that the company should appraise all future investment projects using the dual Stern-Stewart concepts of EVA and MVA.

You are not convinced that substituting value added analyses for the company's existing investment model will contribute anything to its wealth maximisation objective.

**Required:**

Based on your reading of the *SFM* text

1. Outline how a company maximises the NPV of all its projects as a basis for shareholder wealth maximisation.
2. Present the *three* Stern-Stewart equations required to prove the inter-relationship between value added and NPV.
3. Manipulate these equations to illustrate whether the Stern-Stewart model is financially equivalent to NPV maximisation.
4. Summarise your thoughts on the case for value added.

**An Indicative Outline Solution**

Throughout most of our text and exercises we have assumed that companies *should* maximise wealth using the NPV investment model and optimum financing, a combination of which maximises cash inflows at minimum cost. We can summarise the approach as follows.

1. **NPV Maximisation and Shareholder Wealth**

   A NPV project calculation requires the derivation of a discount rate, based upon the mathematical concept of a *weighted average* to formulate a company's *WACC* as an appropriate cut-off rate for investment. For example, with only two sources of capital (equity and debt say) and using our standard notation, a general formula for WACC is given by:

$$K = K_e (V_E / V) + K_d (V_D / V)$$

Computationally, the component costs of capital are weighted as a proportion of the company's total market value and the results summated (i.e. added together).

We can then derive any project's NPV by discounting its cash flow series at the company's WACC (i.e. K) and subtracting the cost of the investment.

$$[(PV@WACC - I_0] = NPV$$

Now assume that the normative objective of our company (Grohl) is to maximise shareholder wealth. The NPV maximisation approach to investment appraisal means that if a choice must be made between alternatives (because projects are mutually exclusive, or capital is rationed) the highest NPV should be selected, subject to a comparison of their risk-return profiles using mean variance analysis.

To maximise NPV, it is also the company's responsibility to acquire capital from various sources in the most efficient (least-costly) manner to establish an overall discount rate. The derivation of this marginal WACC (whether it be traditional or MM based) should represent the optimum discount rate. The project which then produces the highest return in excess of this WACC should therefore maximise NPV and not only exceed shareholders' expectations of a dividend or capital gain but also the returns required by all other providers of capital.

2. **The Value Added Equations**

Optimum investment and financial decision models employed by financial managers under risk and non-risk conditions should maximise corporate wealth through the inflow of cash at minimum cost. It is a basic tenet of financial theory that the NPV maximisation of all a firm's projects satisfies this objective. So, what does the Stern-Stewart model offer the Grohl company, over and above the universally accepted NPV decision rule?

According to Stern-Stewart, economic value added (EVA) is a periodic, incentive-based earnings performance driver that is correlated to increased shareholder value measured by market value added (MVA). Whilst Stern-Stewart's precise derivation of value added has remained highly *secretive* since they adopted it as their own in 1982 (perhaps explaining, why it has captured the corporate imagination and attracted media comment world-wide) the concept has a long academic and empirical pedigree. Like much else in finance, it can be traced back to the "golden age" of the 1960s.

The economic rationale for the Stern-Stewart model is best explained to the Grohl Company by reference to Chapter Eight of the *SFM* text, which defines all the constituent components, notation and purpose of the following three equations:

EVA = NOPAT (free cash flow) less the *money* cost of total capital investment = NOPAT – C.K

MVA = Market value less total capital = V – C

V = Market value = Capital plus the present value of all future EVA = C + PV(EVA)

3. **The Financial Equivalence between Value Added and NPV Maximisation**
   The inter-relationship between the Stern-Stewart model, NPV maximisation and shareholder wealth can now be explained by manipulating the relationships between the previous three equations asfollows. Given:

   1. EVA = NOPAT – C.K
   2. MVA = V – C
   3. V = C + PV(EVA)

   First take the difference between Equations (2) and (3) to redefine MVA.

   4. MVA = PV (EVA)

   Next, because the EVA equation represents a current cash surplus after subtracting the money cost of capital investment from NOPAT (what Stern-Stewart term *free cash flow*) it must equal the NPV of all a firm's projects if they are discounted using K as a common WACC.

   According to Stern-Stewart the MVA of Equation (4) may be redefined as follows

   5. MVA = PV of all future EVA = S NPV

4. **Summary**
   Your initial dispute with the Finance Director of the Grohl Company who recommends the substitution of value added concepts for NPV to pursue shareholder maximisation appears justifiable. Theoretically, the two models should be *financially equivalent*, so why change over?

Presumably, the Finance Director's preference for value added reflects the views of Stern-Stewart. Because management do not provide project NPVs based on *internal* cash flow data for *external* users of published accounts, what markets require is an *equivalent* model, which they can derive from the data actually contained in those accounts. Only then can investors assess corporate performance on the same terms that management initially justified project decisions on their behalf. According to Stern-Stewart, value added provides such a measure and as a consequence, it also acts as a *control* on dysfunctional management behaviour (the agency principle).

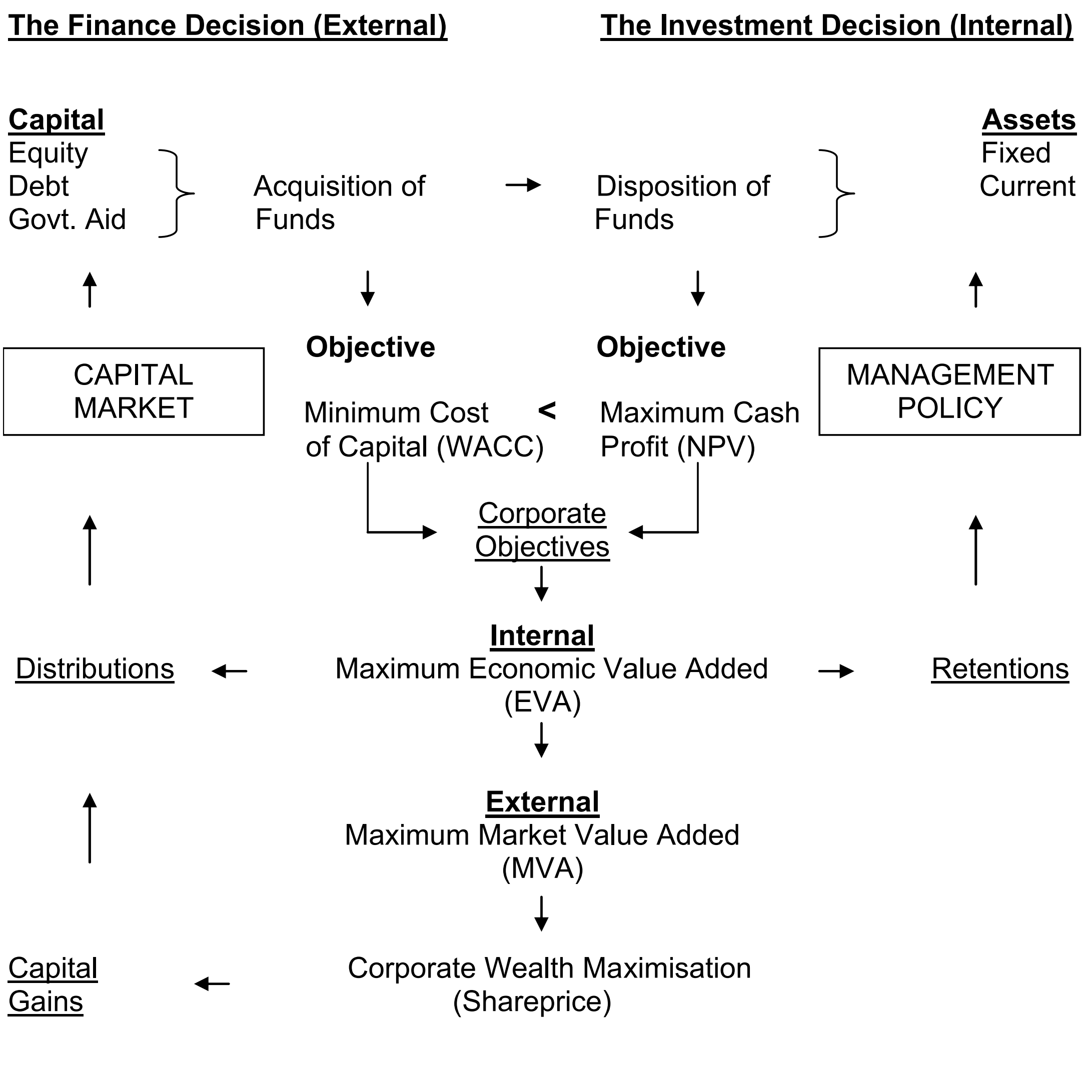


**Figure 8.1:** Strategic Financial Management and the Stern-Stewart Model

Figure 8.1 (reproduced from the *SFM* text) illustrates how the Stern-Stewart model should fit seamlessly into a managerial framework of *internal* NPV analyses and *external* shareholder wealth maximisation. However, there are still nagging doubts concerning its practical application.

A long-standing criticism is that because the Stern-Stewart consultancy is secretive (for sound commercial reasons) it does make it difficult to verify their claims. For example, the EVA formula *de-leverages* published post-tax accounting profits to derive NOPAT based on numerous cash flow adjustments that are not in the public domain. And even where the value added computation of public companies is transparent, it is rarely measured in the same way (see Weaver 2001).

Prior to the current wave of financial crises and market volatility, which now makes *trend* research difficult, Griffith (2004) also sampled the EVA figures of 63 corporate consultancy clients available on the Stern-Stewart web page at www.sternstewart.com. He confirmed too, that neither EVA, nor MVA, were good indicators of performance.

Even if the profitability side of the EVA equation corresponds to the periodic cash flow of NPV calculations, (*free cash flow* explained in Chapter Eight of *SFM*) a fundamental problem remains. How do Stern-Stewart measure the WACC (i.e. K) in the third term of their formula?

$$EVA = NOPAT - C.K$$

EVA calculations, like NPV, are based on the common assumption that an optimum WACC (central to the finance function outlined in Figure 8.1) can be satisfactorily defined, either as a money cost of capital in the previous equation, or the NPV discount rate (r =K) in the following equation.

$$NPV = [\sum_{t=1}^{n} C_t / (1+K)^t] - I_0$$

However, as we observed in Chapter Seven there are two schools of thought. The *traditional* approach to investment finance subscribes to a “pecking order” framework.

WACC falls with leverage because firms prefer cheaper internal to external financing and then cheaper debt to equity, if they need to issue financial securities to support their investment.

Alternatively, we have the MM hypothesis.

WACC is constant, irrespective of leverage, because any change in the gearing ratio produces a compensatory change in the cost of equity to counter the change in the level of financial risk.

## Exercise 8.2: Current Issues and Future Developments

Whilst the value added debate continues, it is worth noting that the Stern-Stewart model does provide support for the MM capital structure hypothesis. Both of their WACC derivations are driven by earnings (business risk). By implication, Stern-Stewart must also support the MM dividend hypothesis that financial risk is irrelevant.

Perhaps you recall from previous chapters that the MM capital structure and dividend irrelevancy theories are entirely consistent with one another. Based on their economic “law of one price” and “perfect substitution”:

- Personal (home made) leverage is equivalent to corporate leverage.
- Capital gains (home made dividends) are equivalent to corporate dividends.

It therefore seems reasonable to assume that if Stern-Stewart accept the MM dividend irrelevancy hypothesis:

Value added is dependent upon investor agreement on the level of de-leveraged post-tax earnings (NOPAT. or what MM term NOI) and their degree of business risk, rather than the financial risk associated with proportion distributed.

If the dividend-retention decision is irrelevant to the marketpricing of shares, then so too, is the division of returns between debt and equity, which determines WACC (K) in the EVA equation.

So, let us conclude our analysis of the value added concept by illustrating the relationship between the capital structure and dividend irrelevancy hypotheses of MM. Both underpin the Stern-Stewart model and remain at the heart of modern financial management (summarised in Figure 8.1).

For the purposes of uniformity, we shall ignore the tax deductibility of debt. This follows logically from our analysis of MM's basic propositions in Chapter Seven. Besides, if their theory fails the test at a rudimentary level of logic, why bother with greater realism?

Consider the Edge Company, an all-equity firm financed by 100,000 £1 shares (nominal). Total earnings are £100,000 and the market price per share is £10.00.

| Using familiar notation from previous chapters: | | | | £ |
|---|---|---|---|---|
| Earnings | | $E_1$ | = | 100,000 |
| Equity: | | | | |
| | Market value | $V_{E1}$ | = | 1,000,000 |
| | Capitalisation rate | $K_{e1} = E_1 / V_{E1}$ | = | 10% |
| Total value | | $V_U = V_{E1}$ | = | 1,000,000 |

Now consider the Bono Company, an identical firm in terms of business risk with the same level of earnings. It differs only in the manner by which it finances its operations. 50 per cent of the market value of capital is represented by bonds that yield 5 per cent. According to MM, because identical assets cannot sell at different prices in the same market (i.e. total corporate value and the price per share are the same) it follows that:

| | | | | |
|---|---|---|---|---|
| Earnings | | $E_2$ | = | 100,000 |
| Debt : | | | | |
| | Market value | $V_D$ | = | 500,000 |
| | Interest (£) | $I = K_d V_D$ | = | 25,000 |
| | Interest (%) | $K_d = I / V_D$ | | = 5% |
| Equity: | | | | |
| | Market value | $V_{E2}$ | = | 500,000 |
| | Capitalisation rate | $K_{e2} = (E_2 - K_d V_D) / V_{E2}$ | = | 75,000 / 500,000 |
| | | | = | 15% |
| Total value | | $V_G = V_{E2} + V_D$ | | = 1,000,000 |

**Required:**

Based on your reading of the *SFM* text and previous exercises:

1. Derive the WACC for Edge and Bono respectively.
2. Explain the implications of your findings.

**An Indicative Outline Solution**

In previous exercises we observed that if management maximise shareholder wealth, using either EVA or NPV decision models, their optimal financial policy should represent a uniform, least-cost combination of debt and equity that maximises cash inflows at minimum cost.

1. **The Derivation of a Uniform WACC**
   We can derive the WACC for both firms using either of the following general formulations.

$$K = K_e (V_E / V) + K_d (V_D / V)$$

$$K = K_e (W_E) + K_d (W_D)$$

(where $W_E$ and $W_D$ represent the weightings applied to equity and debt respectively).

Now, let us apply the data to the previous equations, where $K_u$ and $K_g$ represent the WACC for the ungeared and geared firm respectively.

$$\text{Edge } K_u = (10\% \times 1.0) = 10\%$$
$$\text{Bono } K_g = (15\% \times 0.5) + (5\% \times 0.5) = 10\%$$

Thus, irrespective of gearing, the WACC for both companies is identical.

2. **The Implications**

The previous analysis follow logically from the MM *arbitrage* proof in our last chapter. The equity capitalisation rate has risen with gearing to offset *exactly* the lower cost of debt, which also explains MM's proposition that two identical assets (shares and corporate value in our example) cannot exhibit different prices. Consequently, the WACC or cut-off rate for investment for any firm in a particular class of business risk equals the equity capitalisation rate for an all-equity firm in that class. In general terms if:

$$V_{EU} = V_U = V_G$$

It follows that:

$$K_{eu} = K_u = K_g$$

So, given the MM hypotheses that the market value of investment is independent of a company's financial policy (because dividend-retention and debt-equity ratios are *perfect economic substitutes*) the Stern-Stewart model should confirm that a company's overall cost of capital subtracted from its income and hence market value is divorced from its gearing.

## Summary and Conclusions

Our study of finance began with an *idealised* picture of rational, risk-averse investors. They should formally analyse the profitability of one course of action in relation to another in pursuit of their wealth maximisation objectives. In a sophisticated mixed economy outlined in Figure 8.2 where the ownership of companies is divorced from control (the *agency* principle), we then defined the strategic, *normative* goal of financial management as follows.

The implementation of investment and financing decisions using risk-adjusted wealth maximising techniques, such as expected net present value (ENPV) and certainty equivalents, which generate money profits in the form of retentions and distributions that satisfy the firm's *owners* (a multiplicity of ordinary shareholders) thereby maximising share price.

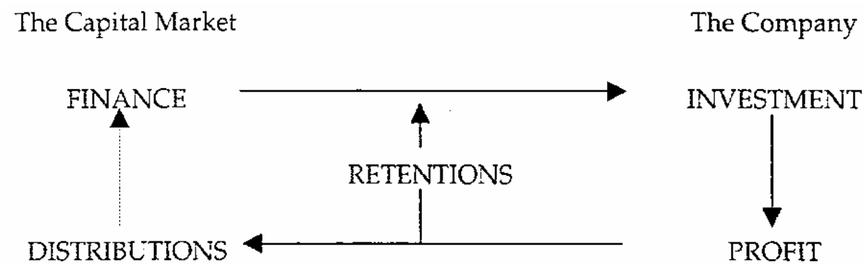


**Figure 8:2:** The Mixed Market Economy

You will recall that if firms make money profits that *exceed* their overall cost of funds (a positive NPV) they create what is termed *economic value added* (EVA), which provides a "real" surplus at no expense to their stakeholders. In a *perfect capital market with no barriers to trade*, demand for a company's shares, driven by its EVA, should then exceed supply. Share price will rise, thereby creating *market value added* (MVA) for the mutual benefit of the firm, its owners and prospective investors.

Of course, the price of shares can fall, as well as rise, depending on economic circumstances. Companies engaged in inefficient or irrelevant activities, which produce losses (negative NPV and EVA) are gradually starved of finance because of reduced dividends, inadequate retentions and the capital market's unwillingness to replenish their asset base at current market prices (negative MVA).

Figure 8.3 distinguishes the "winners" from the "losers" in their drive to add value by summarising why some companies fail. These may then fall prey to take-over as share values plummet (or they may even go into liquidation).

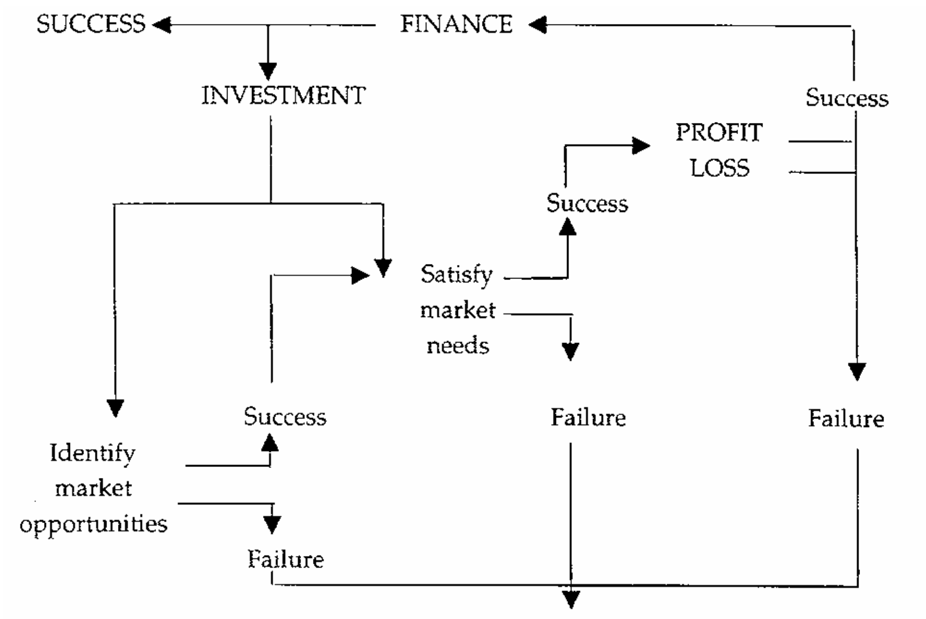


**Figure 8.3:** Corporate Economic Performance: Winners and Losers

Throughout the remainder of the text, we defined successful management policies of wealth maximisation, which increase share price, in terms of two distinct but nevertheless *inter-related* functions

- The *investment function*, which identifies and selects a portfolio of investment opportunities that *maximise* anticipated net cash inflows commensurate with risk.
- The *finance function*, which identifies potential fund sources (internal and external, debt or equity, long or short) required to sustain investments, evaluates the risk-adjusted return expected by each, then selects the optimum mix that will *minimise* their overall weighted average cost of capital (WACC).

The managerial investment function and finance function are linked by the company's WACC. You will recall that from a financial perspective, it represents the overall cost incurred in the acquisition of funds. A complex concept, it not only concerns explicit interest on borrowings or dividends paid to shareholders. Companies also finance their operations by utilising funds from a variety of sources, both long and short term, at an implicit or opportunity cost, notably retained earnings, without which companies would presumably have to raise funds elsewhere. In addition, there are implicit costs associated with depreciation and other non-cash expenses. These too, represent retentions that are available for reinvestment.

Finally, in terms of the corporate investment decision, we reconciled the NPV maximisation of all a firm's projects with EVA and MVA maximisation using WACC as an appropriate cut-off rate for investment.

In our *ideal world* characterised by rational investors and perfect markets, the strategic objectives of financial management relative to the investment and finance decisions that enhance shareholder wealth can be characterised as follows:

Funds from any source should only be invested in capital projects if their marginal yield at least equals the rate of return that the finance provider can earn elsewhere on comparable investments of equivalent risk.

Cash profits should then exceed the overall cost of investment (WACC) producing either a positive NPV or EVA which can either be distributed as a dividend or retained to finance future investments.

If management wish to increase shareholder wealth, (MVA) using share price as a *vehicle*, then earnings (measured by NPV or EVA) rather than dividends should be the *driver*.

So, there you have it. An introduction to strategic financial management based on established theories. But as we observed in Chapter One of this and the *SFM* companion text, such theories attract legitimate criticism in a world that is *far from ideal* characterised by geo-political and economic instability, financial meltdown and recession.

So, whilst the exercises presented in this text support a sturdy framework for the analysis of investment and finance decisions, it remains to be seen whether it is a "castle built on sand".